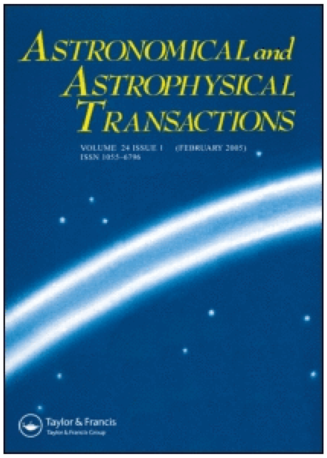

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Sp-branes: integrable multidimensional cosmologies

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Sp-branes: integrable multidimensional cosmologies

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We investigate time-dependent solutions (Sp-brane solutions) for product manifolds consisting of factor spaces where only one of them is of a non-Ricci-flat type. Our model contains a minimally coupled free scalar field and form field (flux) as matter sources. We discuss the possibility of generating late-time acceleration of the Universe. For these models, we investigate the variation with time of the effective four-dimensional fundamental 'constants'. We show that experimental bounds for the fundamental constant variations apply strong restrictions to the considered models.

Keywords: Multidimensional cosmology; Acceleration; Variation in fundamental constants

1. Introduction

Recent observational data provide strong evidence in favour of the accelerating expansion of our Universe which began approximately at red shift $z \approx 1$ and continues until the present time. The explanation of this acceleration is one of the main challenges of modern cosmology. Among a number of attempts, the models originating from fundamental theories (e.g. string or M-theory) are of the most interest. For example, it was shown that some space-like brane (Sp brane) solutions have stages of accelerating expansion [1–9]. The topology of these models represents a product manifold which consists of a number (usually two) factor spaces that behave dynamically with time. One of the factor spaces (Sp brane) corresponds to the external (our) space which undergoes the stage of accelerating expansion. Another factor space corresponds to the dynamic internal space. It is well known that the dynamics of internal spaces results in variations (dynamic behaviour) in the fundamental 'constants' of an effective 4D theory. (As we analyse the model in the Einstein frame, we consider only variations in the fine-structure 'constant' which is inversely proportional to the volume of the internal space.) However, there are strong experimental bounds for the variations in the fundamental constants (see [10] and references therein). Thus, the dynamic behaviour of the internal space should be slow enough to satisfy these restrictions.

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In the present paper, we investigate the Sp brane solution with flux (form field) and a minimally coupled free scalar field as matter sources. Here, the external space is a Ricci flat and the internal space may have any sign of curvature: spherical, hyperbolic or Ricci flat. We analyse the dynamic behaviour of this model, looking for accelerating stages for the external space. Then, we check whether these solutions do satisfy the experimental bounds for variations in the fine-structure constant or not. We show that the external space has stages of accelerating expansion irrespective of the sign of the internal space curvature. However, variations in the fine-structure constant do not satisfy the experimental bounds.

2. General set-up

We consider a cosmological model with the factorizable metric

$$g = \Omega^2 \left[-dt^2 + a_0^2(\tau) g^{(0)}\right] + a_1^2(\tau) g^{(1)} \tag{1}$$

defined on the manifold with the product topology $M = R \times R^{d_0} \times M^{d_1}$, where R^{d_0} is the d_0-dimensional Ricci-flat external (our) space with the metric $g^{(0)}$ and the scale factor a_0 and where $g^{(1)}$ and a_1 are the metric and the scale factor, respectively, for the internal space M^{d_1}. The internal space is an Einstein space with curvature $R_1 = kd_1(d_1 - 1)$, $k = -1, 0, 1$. The total number of dimensions is $D = 1 + d_1 + d_0$. The metric (1) is a metric in the synchronous time gauge in the Einstein frame, where $\Omega(\tau) = a_1^{-d_1/(d_0-1)}$.

The action for the considered model is

$$S = \frac{1}{2\kappa^2} \int_M \mathrm{d}^D x \, (|g|)^{1/2} \left(R[g] - g^{MN} \partial_M \varphi \partial_N \varphi - \frac{1}{2 \times d_1!} F^2_{[d_1]} \right), \tag{2}$$

where φ is a free minimally coupled homogenous scalar field and $F_{[d_1]}$ is the d_1 form-field strength which is taken as $F_{[d_1]} = b \,\mathrm{vol}\left[M^{d_1}\right] \to F^2_{[d_1]} = b^2 d_1! a_1^{-2d_1}$. Minimizing this action, we obtain the following solution (the general method for this kind of model was described in full in [11–14]):

$$a_0(\tau) = \left(\mathrm{e}^{q_2 v^0}\, \mathrm{e}^{-(q_2/q_1) v^1}\right)^{d_1/[(d_0-1)(d_1-1)]}, \tag{3}$$

$$a_1(\tau) = \left(\mathrm{e}^{q_2 v^0}\, \mathrm{e}^{-q_1 q_2 v^1}\right)^{1/d_1 - 1}, \tag{4}$$

$$\varphi(\tau) = p^2 \tau, \tag{5}$$

where

$$v^0 = \begin{cases} q_2^{-1} \left\{ \ln\left[\left(\dfrac{2\varepsilon}{R_1}\right)^{1/2}\right] - \ln\left\{\cosh\left[(q_2 2\varepsilon)^{1/2}\tau\right]\right\}\right\}, & R_1 > 0, \\ q_2^{-1} \left\{ \ln\left[\left(\dfrac{2\varepsilon}{|R_1|}\right)^{1/2}\right] - \ln\left\{\sinh\left[(q_2 2\varepsilon)^{1/2}\tau\right]\right\}\right\}, & R_1 < 0, \\ (2\varepsilon)^{1/2}\tau, & R_1 = 0,\ p^0 = \pm(2\varepsilon)^{1/2}, \end{cases} \tag{6}$$

$$v^1 = (q_1 q_2)^{-1} \left(\ln\left|\frac{p^1}{b}\right| - \ln\left\{\cosh\left[q_1 q_2 p^1 (\tau - \tau_0)\right]\right\}\right).$$

Here, $q_2 = ((d_1 - 1)/d_1)^{1/2}$, $q_1 = (d_0 d_1/(D-2))^{1/2}$, τ_0, ε, p^1 and p^2 are the constants of integration with the following constraint: $2\varepsilon = (p^1)^2 + (p^2)^2$. This solution is obtained in the harmonic time gauge. The relation between the harmonic time τ and synchronous time t is $\mathrm{d}t = f(\tau)\mathrm{d}\tau$, where $f(\tau) = a_0^{d_0}$.

In what follows, to investigate the accelerating behaviour of the external space as well as the variations in the fine-structure constant, we shall need the deceleration parameter for the external space given by

$$\begin{aligned} -q_0 &= \frac{1}{a_0(t)}\frac{\mathrm{d}^2 a_0(t)}{\mathrm{d}t^2} \\ &= \frac{1}{f(\tau)^2 a_0(\tau)}\left(\frac{\mathrm{d}^2 a_0(\tau)}{\mathrm{d}\tau^2} - \frac{1}{f(\tau)}\frac{\mathrm{d}f(\tau)}{\mathrm{d}\tau}\frac{\mathrm{d}a_0(\tau)}{\mathrm{d}\tau}\right) \\ &= \frac{1}{f(\tau)^2 q_2(d_1-1)}\left[\left(\ddot{v}^0 - \frac{1}{q_1}\ddot{v}^1\right) - \frac{1}{q_2^2}\left(\dot{v}^0 - \frac{1}{q_1}\dot{v}^1\right)^2\right] \end{aligned} \tag{7}$$

and the Hubble parameters for both factor spaces given by

$$H_i = \frac{1}{a_i(t)}\frac{\mathrm{d}a_i(t)}{\mathrm{d}t} = \frac{1}{f(\tau)a_i(\tau)}\frac{\mathrm{d}a_i(\tau)}{\mathrm{d}\tau}, \quad i = 0, 1. \tag{8}$$

As we mentioned above, the effective 4D fine-structure constant is inversely proportional to the volume of the internal space: $\alpha \propto V_1^{-1} \propto a_1^{-d_1}$. There are strong constraints on $\dot{\alpha}/\alpha$ [10]. For our calculation we take $|\dot{\alpha}/\alpha| \lesssim 10^{-15}\,\mathrm{year}^{-1}$, which follows from observations of the spectra of quasars. So, we can write the following estimate:

$$\left|\frac{\dot{\alpha}}{\alpha}\right| = \left|\frac{\dot{V}_1}{V_1}\right| = |d_1 H_1| \leqslant 10^{-15}\,\mathrm{year}^{-1}. \tag{9}$$

Combining this with the accepted value for the current Hubble rate $H_0 = \dot{a}_0/a_0 \approx 10^{-10}\,\mathrm{year}^{-1}$ leads to

$$\left|\frac{H_1}{H_0}\right| = \left|\frac{d_0-1}{d_1}\frac{\dot{v}^0 - q_1\dot{v}^1}{\dot{v}^0 - q_1^{-1}\dot{v}^1}\right| \leqslant 10^{-5}. \tag{10}$$

Let us now test solutions (3)–(6) for different signs of the curvature of the internal space.

3. Spherical internal space

First, we consider the case with spherical internal space ($R_1 > 0$). From equations (7) and (8) we obtain the conditions of the expansion and acceleration of the external space respectively:

$$-(2\varepsilon)^{1/2}\tanh\left[q_2(2\varepsilon)^{1/2}\tau\right] + \frac{1}{q_1}p^1\tanh[q_1 q_2 p^1(\tau-\tau_0)] > 0 \tag{11}$$

and

$$\begin{aligned} &-\frac{1}{d_0-1}2\varepsilon\tanh^2[q_2(2\varepsilon)^{1/2}\tau] - \left(1 + \frac{1}{q_1^2 q_2^2}\right)(p^1)^2\tanh^2[q_1 q_2 p^1(\tau-\tau_0)] - (p^2)^2 \\ &+ \frac{2}{q_1 q_2^2}p^1(2\varepsilon)^{1/2}\tanh[q_2(2\varepsilon)^{1/2}\tau]\tanh[q_1 q_2 p^1(\tau_1-\tau_0)] > 0. \end{aligned} \tag{12}$$

Simple analysis of these conditions shows that the accelerating expansion of the external space takes place for $\tau_0 < 0$ and for the corresponding choice of the range of values p^2. Here, the stage of expansion is split into three successive periods: deceleration, acceleration and again deceleration. Then, the decelerating stage of contraction follows the stage of expansion.

The ratio of Hubble parameters (10) for this particular model is

$$\frac{H_1}{H_0} = \frac{d_0 - 1}{d_1} \frac{-(2\varepsilon)^{1/2} \tanh[q_2 (2\varepsilon)^{1/2}\tau] + q_1 p^1 \tanh[q_1 q_2 p^1 (\tau - \tau_0)]}{-(2\varepsilon)^{1/2} \tanh[q_2 (2\varepsilon)^{1/2}\tau] + q_1^{-1} p^1 \tanh[q_1 q_2 p^1 (\tau - \tau_0)]} \approx 1. \tag{13}$$

It is clear that this model does not satisfy the constraint (10) with the exception of a short period near τ_1 where

$$(2\varepsilon)^{1/2} \tanh[q_2 (2\varepsilon)^{1/2}\tau_1] = q_1 p^1 \tanh[q_1 q_2 p^1 (\tau_1 - \tau_0)].$$

4. Hyperbolic internal space

Now, we consider the case with hyperbolic internal space ($R_1 < 0$). We shall analyse the behaviour of the model for $\tau < 0$, because at the point $\tau = 0$ the function $v^0(\tau)$ is divergent. This choice of the range of τ does not affect the results of our consideration because the dynamic pictures for both $\tau < 0$ and $\tau > 0$ are equivalent.

The conditions of the expansion and acceleration of the external space are

$$-(2\varepsilon)^{1/2} \coth[q_2 (2\varepsilon)^{1/2}\tau] + \frac{1}{q_1} p^1 \tanh[q_1 q_2 p^1 (\tau - \tau_0)] > 0 \tag{14}$$

and

$$\begin{aligned} &- \frac{1}{d_0 - 1} 2\varepsilon \coth^2[q_2 (2\varepsilon)^{1/2}\tau] - \left(1 + \frac{1}{q_1^2 q_2^2}\right) (p^1)^2 \tanh^2[q_1 q_2 p^1 (\tau - \tau_0)] - (p^2)^2 \\ &+ \frac{2}{q_1 q_2^2} p^1 (2\varepsilon)^{1/2} \coth[q_2 (2\varepsilon)^{1/2}\tau] \tanh[q_1 q_2 p^1 (\tau - \tau_0)] > 0, \end{aligned} \tag{15}$$

respectively. From these inequalities it follows that the external space has a stage of accelerating expansion for an arbitrary sign of τ_0 and for a proper choice of the range of the parameter p^2.

The ratio (10) of the Hubble parameters in this case is

$$\frac{H_0}{H_1} = \frac{d_0 - 1}{d_1} \frac{-(2\varepsilon)^{1/2} \coth[q_2 (2\varepsilon)^{1/2}\tau] + q_1 p^1 \tanh[q_1 q_2 p^1 (\tau - \tau_0)]}{-(2\varepsilon)^{1/2} \coth[q_2 (2\varepsilon)^{1/2}\tau] + q_1^{-1} p^1 \tanh[q_1 q_2 p^1 (\tau - \tau_0)]} \approx 1. \tag{16}$$

As we can see, the model with the accelerating Ricci-flat external space and hyperbolic internal space cannot satisfy the experimental bounds (10) with the exception of a short period near the point τ_1, where

$$(2\varepsilon)^{1/2} \coth[q_2 (2\varepsilon)^{1/2}\tau_1] = q_1 p^1 \tanh[q_1 q_2 p^1 (\tau_1 - \tau_0)].$$

5. Ricci-flat internal space

Let us now investigate the case of the Ricci-flat internal space $R_1 = 0$. It can be easily seen that in this case the condition of the expansion of the external space given by

$$(2\varepsilon)^{1/2} + \frac{1}{q_1} p^1 \tanh[q_1 q_2 p^1(\tau_1 - \tau_0)] > 0 \tag{17}$$

is satisfied for an arbitrary value of τ. For the acceleration condition we have

$$\begin{aligned} &-\left(1 + \frac{1}{q_1^2 q_2^2}\right)\left(\frac{p^1}{(2\varepsilon)^{1/2}}\right)^2 \tanh^2[q_1 q_2 p^1(\tau_1 - \tau_0)] \\ &\quad - \frac{2}{q_1 q_2^2}\frac{p^1}{(2\varepsilon)^{1/2}} \tanh[q_1 q_2 p^1(\tau_1 - \tau_0)] + \left[\left(\frac{p^1}{(2\varepsilon)^{1/2}}\right)^2 - \frac{1}{q_2^2}\right] > 0. \end{aligned} \tag{18}$$

On the left-hand side of this inequality we have a quadratic polynomial with respect to tanh (with a negative sign of the higher-degree term). Obviously, this polynomial can have positive values only if its determinant

$$D = 4\frac{(p^1)^2}{2\varepsilon}\left[\frac{(p^1)^2}{2\varepsilon}\left(1 + \frac{1}{q_1^2 q_2^2}\right) - \frac{1}{q_2^2}\right] \tag{19}$$

is also positive. Thus, we arrive at the following condition for the acceleration:

$$\frac{(p^1)^2}{2\varepsilon} > \frac{d_0 d_1}{d_0 d_1 + d_1 - 1}. \tag{20}$$

Keeping in mind that $2\varepsilon = (p^1)^2 + (p^2)^2$, we can easily obtain from this expression the range of p^2 which allows acceleration of the external space.

However, analysing the fine-structure constant variations, we obtain

$$\frac{H_0}{H_1} = \frac{d_0 - 1}{d_1}\frac{(2\varepsilon)^{1/2} - q_1 p^1 \tanh[q_1 q_2 p^1(\tau_1 - \tau_0)]}{(2\varepsilon)^{1/2} - q_1^{-1} p^1 \tanh[q_1 q_2 p^1(\tau_1 - \tau_0)]} \approx 1, \tag{21}$$

which clearly shows that the experimental bounds are not satisfied in this model. These constraints hold true only about a point τ_1 satisfying the equation

$$(2\varepsilon)^{1/2} = q_1 p^1 \tanh[q_1 q_2 p^1(\tau_1 - \tau_0)].$$

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