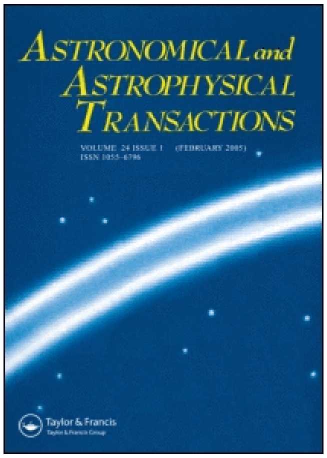

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Effective Friedmann model from multidimensional cosmologies

A. Zhuk [a]
[a] Department of Theoretical Physics and Astronomical Observatory, Odessa National University, Odessa, Ukraine

Online Publication Date: 01 October 2006
To cite this Article: Zhuk, A. (2006) 'Effective Friedmann model from multidimensional cosmologies', Astronomical & Astrophysical Transactions, 25:5, 435 - 440
To link to this article: DOI: 10.1080/10556790601119368
URL: http://dx.doi.org/10.1080/10556790601119368

Astronomical and Astrophysical Transactions
Vol. 25, Nos. 5–6, October–December 2006, 435–440

Effective Friedmann model from multidimensional cosmologies

A. ZHUK*

Department of Theoretical Physics and Astronomical Observatory, Odessa National University,
2 Dvoryanskaya Street, Odessa 65026, Ukraine

(Received 13 October 2006)

We investigate the possibility of the construction of the conventional Friedmann cosmology for our observable Universe if the underlying theory is the multidimensional Kaluza–Klein model. We show that the effective Friedmann model obtained by dynamic compactification of the multidimensional model is faced with too strong variations in the fundamental constants. On the other hand, models with stable compactification of the internal space are free from this problem and also result in conventional four-dimensonal cosmological behaviour for our Universe. We prove a no-go theorem, which shows that stable compactification of the internal spaces is possible only if the equations of state in the external and internal spaces are properly adjusted to each other. With a proper choice of parameters (fine tuning), the effective cosmological constant in this model provides the late-time acceleration of the Universe.

Keywords: Multidimensional cosmological models; Dynamic compactification; Stable compactification; Fundamental constant variation

1. Introduction

The multidimensionality of our Universe is one of the most intriguing assumptions in modern physics. It is a natural ingredient of theories which unify different fundamental interactions with gravity, such as the string or M-theory. However, introduction of extra dimensions results in the complex dynamic behaviour of the multidimensional Universe. This deviation from evolution of the Friedmann–Robertson–Walker (FRW) Universe may have dramatic consequences and contradict observable data. The main purpose of this paper is to investigate the possibility of the conventional description of effective four-dimensional (4D) cosmological models obtained from the Kaluza–Klein (KK) models.

We investigate the KK models where space–time is endowed with a multicomponent perfect fluid [1–4]. Within the standard KK models without branes and according to the present level of the experimental data, the internal spaces are unobservable if their scales are of the order of or less than the Fermi length $L_{\mathrm{F}} \approx 10^{-17}$ cm $\approx 1\ \mathrm{TeV}^{-1}$. Such small scales can be achieved in

*Email: zhuk@paco.net

Astronomical and Astrophysical Transactions
ISSN 1055-6796 print/ISSN 1476-3540 online
http://www.tandf.co.uk/journals
DOI: 10.1080/10556790601119368

two ways. Firstly, the internal dimensions behave dynamically such that their size decreases below L_{F}. Here, the internal spaces undergoes dynamic evolution all the time. This behaviour is called dynamic compactification. Secondly, the internal spaces can be stabilized near some fixed value, e.g. L_{F}. This behaviour is called stable compactification.

For the first class of models (with dynamic compactification), Mohammedi [5] proposed an approach for the reduction of multidimensional models with perfect fluid to effective 4D models which have the form of the conventional cosmology. In the present paper, we develop this model and show that it provides a very interesting gravitational 'constant' tuning effect. In spite of the dynamic behaviour of an effective 4D gravitational 'constant' and the non-conventional dynamics of an effective 4D energy density, their product behaves exactly as in the FRW scenario. More precisely this product enters into the Friedmann equations. Thus, the external space has evolved dynamically in accordance with the standard FRW cosmology. However, the fundamental constants in this model undergo too large variations.

Next, we consider models with stable compactification. It is worth noting that two particular classes of solutions with stable compactification of the internal spaces (for models with perfect fluid) have already been found and were given in our previous paper [6]. In the present paper we prove a no-go theorem according to which the models with perfect fluid do not admit stable compactification in the case of an arbitrary combination of equations of state in the external and internal spaces. There are only two exceptional classes where stable compactification takes place and these classes exactly coincide with those found in [6]. Hence, these classes entirely exhaust all possibilities for stable compactification. We construct a particular model which belongs to these classes and has a Friedmann-like behaviour for the external space (our Universe) during the radiation- and matter-dominated stages and late-time acceleration. However, the parameters of model should be fine tuned to obtain the observable dark energy.

2. General set-up

To start with, let us consider a cosmological model with factorizable geometry

$$g = -\mathrm{e}^{2\gamma(\tau)}\,\mathrm{d}\tau \otimes \mathrm{d}\tau + L_{\mathrm{Pl}}^2\,\mathrm{e}^{2\beta^0(\tau)}\,g^{(0)}(\boldsymbol{x}) + \sum_{i=1}^{n} L_{\mathrm{Pl}}^2\,\mathrm{e}^{2\beta^i(\tau)}\,g^{(i)}(y), \tag{1}$$

which is defined on the direct product manifold $M_0 \times \prod_i^n \mathcal{M}_i$. The external (our) space–time $M_0 = \mathbb{R} \times \mathcal{M}_0$ has dimensions $D_0 = 1 + d_0 = 4$. In order to obtain the effective 4D cosmology in the form of the Friedmann equations, we assume that the factors $\mathcal{M}_i$ are d_i-dimensional Einstein spaces. The quantities $a \equiv L_{\mathrm{Pl}}\,\mathrm{e}^{\beta^0}$ and $b_i \equiv L_{\mathrm{Pl}}e^{\beta^i}$ $(i = 1, \ldots, n)$ describe scale factors of the external and internal spaces respectively. $D' = \sum_{i=1}^n d_i$ is the total number of the internal dimensions.

The action functional for the considered multidimensional models is

$$S = \frac{1}{2\kappa_D^2}\int_M \mathrm{d}^D x(|g|)^{1/2}(R[g] - 2\Lambda_D) + S_{\mathrm{m}}, \tag{2}$$

where S_{m} is the action functional for bulk matter. By analogy with conventional cosmology, the bulk matter is taken in the form of a m-component perfect fluid with the energy–momentum tensor

$$T_N^M = \sum_{c=1}^m T^{(c)M}_{\ \ N} = \mathrm{diag}\left(-\rho^{(c)}, \underbrace{P_0^{(c)}, \ldots, P_0^{(c)}}_{d_0\ \text{times}}, \ldots, \underbrace{P_n^{(c)}, \ldots, P_n^{(c)}}_{d_n\ \text{times}}\right). \tag{3}$$

Together with the equations of state $P_i^{(c)} = (\alpha_i^{(c)} - 1)\rho^{(c)}$, the conservation equations have the simple integrals

$$\rho^{(c)}(\tau) = A^{(c)} a^{-d_0\alpha_0^{(c)}} \times \prod_{i=1}^{n} b_i^{-d_i\alpha_i^{(c)}}, \tag{4}$$

where $A^{(c)}$ are constants of integration.

With respect to the internal spaces, there are two possible scenarios: either they are stably compactified at the present time values $b_{(0)i} \equiv L_{\mathrm{Pl}}\, \mathrm{e}^{\beta_0^i} = \text{constant}$, or there is no such stabilization and b_i remain dynamic functions. In the case of stable compactification, small in homogeneous particle-like excitations $\bar{\beta}^i(x)$ over the constant background β_0^i describe massive scalar particles (gravexcitons) that develop in the external space–time [7, 8].

3. Dynamic compactification

In this section, we investigate the possibility of conventional cosmology in the case of multidimensional models with dynamic behaviour of the internal spaces. More precisely, we consider the case of dimensional stabilization when the scale factors of the internal spaces b_i decrease with time. For simplicity, we consider the case of one internal space: $n = 1$, $b_1 \equiv b$. A perfect fluid is also taken in the one-component form: $c = 1 \Rightarrow \rho^{(1)} \equiv \rho$, $P_0^{(1)} \equiv P_0$, $P_1^{(1)} \equiv P_1$. In [5], one interesting observation was made: if we suppose that the scale factor of the internal space evolves according to the relation $b = B/a^q$, where $B \equiv$ constant and the parameter q satisfies the condition $d_1 q(d_1 q - q - 6) = 0$, then, for the case of the Ricci-flat internal space ($k_1 = 0$), the Einstein equations are reduced to the familiar 4D form

$$\kappa_0^2 \rho_{(4)} = 3H^2 + \frac{3k_0}{a^2} - \Lambda_D, \tag{5}$$

$$\kappa_0^2 P_{(4)} = -2\frac{\ddot{a}}{a} - H^2 - \frac{k_0}{a^2} + \Lambda_D, \tag{6}$$

where $\rho_{(4)} \equiv \rho V_{d_1}$ and $P_{(4)} \equiv [P_0 - (d_1 q/3)(\rho + P_1)]V_{d_1}$ are the observable 4D energy density and pressure respectively. The volume of the internal space is $V_{d_1} \propto b^{d_1} \propto a^{-d_1 q}$.

Equations (5) and (6) formally reproduce the famous FRW equations (in the presence of the cosmological constant). However, there are two main differences between this effective model and the standard FRW universe. Firstly, the effective 4D gravitational 'constant' $\kappa_0^2 = \kappa_D^2 / V_{d_1}$ is not a constant but a dynamic function. Secondly, the equation of conservation of energy differs from the conventional equation in the FRW Universe in that it has a non-zero right-hand side as given by

$$\frac{\mathrm{d}}{\mathrm{d}t}(a^3 \rho_{(4)}) + P_{(4)} \frac{\mathrm{d}}{\mathrm{d}t}(a^3) = (a^3 \rho_{(4)}) \frac{1}{V_{d_1}} \frac{\mathrm{d}}{\mathrm{d}t}(V_{d_1}) = -(a^3 \rho_{(4)}) d_1 q \frac{\dot{a}}{a}, \tag{7}$$

and it has the following solution:

$$\rho_{(4)} = \rho_0 \left(\frac{a_0}{a}\right)^{3\alpha + d_1 q}. \tag{8}$$

Here, we assume the equation of state $P_{(4)} = (\alpha - 1)\rho_{(4)}$. This behaviour of the energy density differs from the standard behaviour by the additional degree $d_1 q$. However, it is very important to note that the combination $\kappa_0^2 \rho_{(4)} \propto a^{-3\alpha}$ has the conventional form. It follows from the fact that the dynamic behaviour of $\kappa_0^2 \propto a^{d_1 q}$ exactly compensates the additional degree $a^{-d_1 q}$ in the expression for $\rho_{(4)}$. As a result, the dynamic evolution of the Universe in our model exactly coincides with the evolution of the standard FRW Universe. Thus, we recover the standard behaviour of the Universe in our multidimensional model because of the specific dynamics of the effective 4D gravitational constant $\kappa_0^2 \equiv 8\pi G_4 \Rightarrow G_4 \propto V_{d_1}^{-1} \propto a^{d_1 q}$. However, according to the observations (see, for example, [9–11] and references therein), the rate of variations in the effective gravitation constant in this model,

$$\frac{\dot{G}_4}{G_4} \propto H \propto \frac{1}{t} \approx 10^{-10}\,\mathrm{year}^{-1}, \tag{9}$$

should be smaller by at least a further order of magnitude. Moreover, the dynamic behaviour of the internal space results in a variation in the effective 4D fine-structure 'constant'. In our case it results in a similar estimate (9), which is many orders of magnitude greater than follows from observations. Thus, in the considered model we were faced with too strong variations in the fundamental constants.

4. Stable compactification

Let us suppose now that β_0^i, $i = 1, \ldots, n$, defines the position of the stale compactification and $\bar{\beta}^i(x) = \beta^i(x) - \beta_0^i$ are fluctuations over this stably compactified background. It is well known (for details, see [6, 7]) that in a dimensional reduced effective action these fluctuations behave as scalar fields penetrating into our 4D space–time. Their behaviour is defined by the effective potential

$$U_{\mathrm{eff}} = \left(\prod_{i=1}^{n} \mathrm{e}^{d_i \bar{\beta}^i}\right)^{-2/(D_0-2)} \left(-\frac{1}{2}\sum_{i=1}^{n} \tilde{R}_i\, \mathrm{e}^{-2\bar{\beta}^i} + \Lambda_D + \kappa_D^2 \sum_{c=1}^{m} \rho^{(c)}\right), \tag{10}$$

where $\tilde{R}_i := R_i L_{\mathrm{Pl}}^{-2}\,\mathrm{e}^{-2\beta_0^i}$ and $\rho^{(c)}$ is defined by equation (4). Thus, the problem of stabilization of the extra dimensions is reduced now to a search for the minima of the effective potential U_{eff} with respect to the fluctuations $\bar{\beta}^i$:

$$\left.\frac{\partial U_{\mathrm{eff}}}{\partial \bar{\beta}^k}\right|_{\bar{\beta}=0} = 0 \Longrightarrow \tilde{R}_k = -\frac{d_k}{D_0-2}\left(\sum_{i=1}^{n} \tilde{R}_i - 2\Lambda_D\right) + \kappa_0^2 \sum_{c=1}^{m} \rho_{(4)}^{(c)}\left(\xi_k^{(c)} + \frac{2d_k}{D_0-2}\right), \tag{11}$$

where $\rho_{(4)}^{(c)} = \tilde{A}^{(c)}\tilde{a}^{-d_0\alpha_0^{(c)}}$, $\tilde{a}$ is the scale factor of the external space in the Einstein frame and $\xi_i^{(c)} = d_i[\alpha_i^{(c)} - \alpha_0^{(c)} d_0/(d_0-1)]$. The left-hand side of this equation is a constant but the right-hand side is a dynamic function because of the dynamic behaviour of the effective 4D energy density $\rho_{(4)}^{(c)}$. Thus, we arrived at the following theorem.

No-go theorem: *Multidimensional cosmological KK models with the perfect fluid as a matter source do not admit stable compactification of the internal spaces with the exception of the following two special cases:*

(i)

$$\alpha_0^{(c)} = 0, \quad \forall\, \alpha_i^{(c)}, \quad i = 1, \ldots, n, \ \ c = 1, \ldots, m. \tag{12}$$

(ii)

$$\xi_i^{(c)} = -\frac{2d_i}{d_0 - 1} \Longrightarrow \begin{cases} \alpha_0^{(c)} = \dfrac{2}{d_0} + \dfrac{d_0 - 1}{d_0}\alpha^{(c)}, \\ \alpha_i^{(c)} = \alpha^{(c)}, \quad i = 1, \dots, n, \;\; c = 1, \dots, mz. \end{cases} \tag{13}$$

Case (i) corresponds to a vacuum in the external space $\rho_{(4)}^{(c)} = \tilde{A}^{(c)} =$ constant and arbitrary equations of state in the internal spaces. Some bulk matter can mimic such behaviour, e.g. the vacuum fluctuations of quantum fields (the Casimir effect), monopole form fields [7, 12] and the gas of branes [13].

In case (ii), the energy density in the external space is not a constant but a dynamic function with the following behaviour ($d_0 = 3$): $\rho_{(4)}^{(c)}(\tilde{a}) = \tilde{A}^{(c)}\tilde{a}^{-2(1+\alpha^{(c)})}$. For example, in three-dimensional external space, such a perfect fluid has the form of a gas of cosmic strings for $\alpha^{(c)} = 0$, dust for $\alpha^{(c)} = 1/2$ and radiation for $\alpha^{(c)} = 1$.

To build a more or less viable model with stabilized internal spaces, we combine together cases (i) and (ii). To be more precise, additionally to the perfect fluid of type (ii), we endow our model with a monopole form field [7, 12]:

$$U_{\text{eff}} = \underbrace{(\mathrm{e}^{d_1\bar{\beta}^1})^{-2/(D_0-2)}(-1/2\tilde{R}_1\,\mathrm{e}^{-2\bar{\beta}^1} + \Lambda_D + \tilde{f}^2\,\mathrm{e}^{-2d_1\bar{\beta}^1})}_{U_{\text{int}}(\bar{\beta}1)} + \underbrace{\kappa_0^2\sum_{c=1}^{m}\rho_{(4)}^{(c)}(\tilde{a})}_{U_{\text{ext}}(\tilde{a})}, \tag{14}$$

where $\tilde{f}^2 =$ constant is proportional to the strength of the form field. This separation provides a stable compactification of the internal factor space due to the minimum of the first term $U_{\text{int}} = U_{\text{int}}(\bar{\beta}^1)$ as well as the dynamic behaviour of the external factor space due to $U_{\text{ext}} = U_{\text{ext}}(\tilde{a})$ which is exactly the Friedmannian behaviour. The minimum of U_{int} provides us with an effective 4D cosmological constant: $\Lambda_{\text{eff}} := U_{\text{int}}|_{\bar{\beta}^1=0}$.

It can be easily shown that a positive minimum occurs if all parameters are positive and have the same order of magnitude: $\Lambda_{\text{eff}} \sim \tilde{R}_1 \sim \Lambda_D \sim \tilde{f}^2 > 0$. On the other hand, in KK models the size of the extra dimensions at the present time should be $b_{(0)1} \times 10^{-17}\,\text{cm} \approx 1\,\text{TeV}^{-1}$. In this case, $\tilde{R}_1 \propto b_{(0)1}^{-2} \approx 10^{34}\,\text{cm}^{-2}$. Thus, for the effective cosmological constant we obtain a value which is many orders of magnitude greater than the observable value for the dark energy at the present time. The exact expression for $\Lambda_{\text{eff}} = U_{\text{int}}|_{\bar{\beta}^1=0} = -1/2\tilde{R}_1 + \Lambda_D + \tilde{f}^2$ shows that the necessary small value of the effective cosmological constant can be achieved only if the parameters $\tilde{R}_1$, Λ_D and $\tilde{f}^2$ are extremely fine tuned with each other. We see two possibilities for avoiding this problem. Firstly, the inclusion of different forms fields or fluxes may result in a large number of minima (landscape) [14–17] with a sufficiently high probability of finding oneself in a dark-energy minimum. Secondly, we can avoid the restriction $\tilde{R}_1 \propto b_{(0)1}^{-2} \approx 10^{34}\,\text{cm}^{-2}$ if the internal space is Ricci flat: $\tilde{R}_1 = 0$. For example, M_1 can be an orbifold with branes in fixed points [18]. These two possibilities will be developed further in our forthcoming paper.

Acknowledgements

We thank the International Centre for Theoretical Physics (Trieste) for financial support from the Associate Scheme. This research has also been partially supported by the Project on Scientific Cooperation between the Austrian Academy of Sciences and the National Academy of Sciences of Ukraine number 01/04.

References

[1] V.D. Ivashchuk and V.N. Melnikov, Int. J. Mod. Phys. D **3** 795 (1994).
[2] V.D. Ivashchuk and V.N. Melnikov, Classical Quantum Gravity **12** 809 (1995).
[3] A. Zhuk, Classical Quantum Gravity **13** 2163 (1996).
[4] U. Kasper and A. Zhuk, Gen. Relativity Gravitation **28** 1269 (1996).
[5] N. Mohammedi, Phys. Rev. D **65** 104 018 (2002).
[6] U. Günther and A. Zhuk, Classical Quantum Gravity **15** 2025 (1998).
[7] U. Günther and A. Zhuk, Phys. Rev. D **56** 6391 (1997).
[8] U. Günther and A. Zhuk, Phys. Rev. D **61** 124 001 (2000).
[9] J.-P. Uzan, Rev. Mod. Phys. **75** 403 (2003).
[10] P. Loren-Aguilar, E. Garcia-Berro, J. Isern, *et al.*, Classical Quantum Gravity **20** 3885 (2003).
[11] U. Günther, A. Starobinsky and A. Zhuk, Phys. Rev. D **69** 044 003 (2004).
[12] U. Günther, P. Moniz and A. Zhuk, Phys. Rev. D **68** 044 010 (2003).
[13] A. Kaya, J. Cosmology Astroparticle Phys. **0408** 014 (2004).
[14] R. Bousso and J. Polchinski, J. High Energy Phys. **0006** 006 (2000).
[15] L. Susskind, The anthropic landscape of string theory. Available online at: http://arxiv.org/abs/hep-th/0302219.
[16] R. Kallosh and A. Linde, J. High Energy Phys. **0412** 004 (2004).
[17] D. Schwartz-Perlov and A. Vilenkin, J. Cosmology Astroparticle Phys. **0606** 010 (2006).
[18] T. Appelquist, H.-C. Cheng and B.A. Dobrescu, Phys. Rev. D **64** 035 002 (2001).