COSMOLOGICAL PERTURBATIONS IN PRESENCE OF SCALAR FIELDS

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ABSTRACT. We investigate the role of the scalar field assuming its presence at late stages of the Universe evolution together with dust presented by a system of an arbitrary number of gravitating masses (galaxies, groups of galaxies and clusters of galaxies) and the cosmological constant in the framework of the theory of scalar cosmological perturbations. In particular, we discuss the case of the homogeneous scalar field and its possible influence on the gravitational potentials representing metric perturbations. We also focus attention on the interconnection between the astrophysical problem and the cosmological one in the scalar field presence.

Key words: Dark energy, gravitational potential, inhomogeneous Universe, perturbations, phantom, scalar fields, quintessence

1. Introduction

The acceleration of the Universe expansion was discovered little bit more than a decade years ago and was awarding of the Nobel Prize in 2011. There are a large number of attempts to explain the reasons of the acceleration and the most prefable model for today is the ΛCDM model. There the cosmological constant is responsible to answer for the accelerated expansion. However, there are problems connected with the Λ term like the regulation mechanism which can not offset naturally huge vacuum energy down to the cosmologically acceptable value and to solve the conjuction problem of close magnitudes of the non-compensated remnants of the energy density and vacuuam energy of the Universe at present stage of evolution.

But the possibility of the scalar fields introducing as a matter source is well-known and it could help to solve the problems with the cosmological constant.

In previous work [1] we studied scalar fields in the form of perfect fluids. These fluids were considered as barotropic fluids with the linear equation of state (EoS) $p = \omega\rho$ ($\omega < 0$ is the parameter of the equation of state). We showed that, to be compatible with the scalar field perturbations theory, the parameter of the equation of state should be $\omega = -1/3$ and this perfect fluid should be clustered. Such fluids are called quintessence and phantom for $-1 < \omega < 0$ and $\omega < -1$, respectively. Usually they have a time varying parameter ω of the equation of state, but the models with constant ω are also popular for consideration.

In particular, the value $\omega = -1/3$ corresponds to the frustrated network of cosmic strings and $\omega = -2/3$ to the frustrated network of domain walls, but the last one is ruled out for our work.

2. The models with constant ω

The investigation of the viability of the models with constant ω is of interest to answer the question whether such models are an alternative to the cosmological constant. We consider the compatibility of these models with the scalar perturbations of the Friedmann-Robertson-Walker metrics. In the conformal Newtonian gauge, such perturbed metrics is

$$ds^2 \approx a^2 \left[(1+2\Phi)d\eta^2 - (1-2\Psi)\gamma_{\alpha\beta}dx^\alpha dx^\beta\right], \quad (1)$$

where scalar perturbations $\Phi, \Psi \ll 1$ and, following the standart argumentation, we can put $\Phi = \Psi$.

We consider the Universe at late stages of its evolution when galaxies, clusters and groups of galaxies are already formed (also the peculiar velocities of inhomogeneities are much less than the speed of light) and deep inside of the cell of uniformity where the Universe is highly inhomogeneous. The perturbations of perfect fluids with $\omega = const$ are purely adiabatic, i.e. the dissiparive processes are absent. We neglect the

the contribution of radiation, therefore, the Friedmann equations read

$$\frac{3\left(\mathcal{H}^2+\mathcal{K}\right)}{a^2}=\kappa\overline{T}^0_0+\Lambda+\kappa\bar{\epsilon} \tag{2}$$

and

$$\frac{2\mathcal{H}'+\mathcal{H}^2+\mathcal{K}}{a^2}=\Lambda-\kappa\omega\bar{\epsilon}, \tag{3}$$

where $\mathcal{H}\equiv a'/a\equiv(da/d\eta)/a$ and $\kappa\equiv 8\pi G_N/c^4$ (c is the speed of light and G_N is the Newton's gravitational constant), $\overline{T}^{ik}$ is the energy-momentum tensor of the average pressureless dustlike matter.

We show that the only one value $\omega=-1/3$ is possible for the model with constant parameter of the equation of state to be compatible with the scalar perturbations theory [2, 3]. For $\omega=-1/3$ the equation of the gravitational potential and the fluctuation of the energy density of the perfect fluid read, respectively:

$$\Delta\varphi+\left(3\mathcal{K}-\frac{9\pi G_N}{c^4}\varepsilon_0 a_0^2\right)\varphi=4\pi G_N(\rho-\overline{\rho}) \tag{4}$$

and

$$\delta\epsilon=\frac{2\epsilon_0 a_0^2}{c^2a^3}\varphi. \tag{5}$$

We investigate the dependence on the curvature parameter $\mathcal{K}$ and showed that reasonable expressions of the conformal gravitational potential φ exist for any sign of $\mathcal{K}$. If the perfect fluid is absent, the hyperbolic space is preferred.

3. The models with time-varying ω

We consider the case of fluids with the varying parameter ω (e.g., scalar fields with arbitrary potentials). The system of the equations of scalar perturbations, following the standart argumantation $\Phi=\Psi$ and taking into account the possible contributions of the spatial curvature as well as of the dust, read:

$$\Delta\Phi-3\mathcal{H}(\Phi'+\mathcal{H}\Phi)+3\mathcal{K}\Phi=\frac{\kappa}{2}a^2\delta\epsilon_{dust}-$$
$$\frac{\kappa}{2}\left[(\phi_c')^2\Phi-\phi_c'\varphi'-m^2a^2\phi_c\varphi\right], \tag{6}$$

$$\partial_i\Phi'+\mathcal{H}\partial_i\Phi=\frac{\kappa}{2}\phi_c'\partial_i\varphi, \tag{7}$$

$$\frac{2}{a^2}\left[\Phi''+\mathcal{H}\Phi'+\Phi\left(2\frac{a''}{a}-\mathcal{H}^2-\mathcal{K}\right)\right]=$$
$$\kappa\left[-\frac{1}{a^2}(\phi_c')^2\Phi-\phi_c'\varphi'-m^2\phi_c\varphi\right]. \tag{8}$$

Considering of the case $\varphi=0$ gives the result for the potential:

$$V=\frac{\beta^2}{a^2}+C, \tag{9}$$

where $\beta=\phi_c'$.

Taking into account the possible contributions of radiation also, we obtained the Friedmann equations

$$\frac{2\mathcal{H}'+\mathcal{H}^2+\mathcal{K}}{a^2}=\Lambda-\kappa p_{rad}-\kappa p_{\varphi}, \tag{10}$$

$$\frac{3(\mathcal{H}^2+\mathcal{K})}{a^2}=\Lambda+\kappa\epsilon_{dust}+\kappa\epsilon_{rad}+\kappa\epsilon_{\varphi}. \tag{11}$$

From the equation (6), including the contribution of radiation, and the Friedmann equations it follows the equation for definition of the field's fluctuation and gravitational potential.

$$\Delta\Phi-3\mathcal{H}(\Phi'+\mathcal{H}\Phi)+3\mathcal{K}\Phi=$$
$$\frac{\kappa}{2}\frac{\delta\rho c^2}{a}-\frac{\kappa}{2}\left[(\phi_c')^2\Phi-\phi_c'\varphi'-a^2\frac{dV}{d\phi}(\phi_c)\varphi\right]. \tag{12}$$

4. Conclusions

We considered the Universe on the late stage of its evolution and deep inside of the cell of uniformity. Supposing that the Universe contains also the cosmological constant and a perfect fluid with a negative parameter of the equation of state ω, we investigated scalar perturbations of the FRW metrics due to inhomogeneities. We have got that, to be compatible with the theory of scalar perturbations in the case of constant parameter of EoS, this fluid should be clustered and should have the parameter parameter of the equation of state $\omega=-1/3$. It is concentrated around the inhomogeneities and results in screening of the gravitational potential. Such perfect fluid neither accelerates not decelerate the Universe, but the presence of it helps to resolve the Seeliger paradox for any sign of the spatial curvature parameter $\mathcal{K}$. Such perfect fluid can be simulated by scalar fields with the corresponding form of the potentials as well as by the frustrated network of topological defects.

For the scalar fields with the arbitrary potentials (and dust and radiation as additional matter sources) we got the equation for the defenition of the field's fluctuation and gravitational potential. It gives us possibility to study the dynamics of inhomogeneities of the Universe inside of the cell of uniformity.

References

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