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ON A REDUCTION OF NONLINEAR SECOND-ORDER DIFFERENTIAL SYSTEM TO A SOME SPECIAL KIND

Щоголев С. А. Про зведення нелiнiйної диференцiальної системи другого порядку до одного спецiального вигляду. Для нелiнiйної коливної диференцiальної системи другого порядку побудовано перетворення, яке зводить цю систему до близької до системи з повiльно змiнними коефiцiєнтами.
Ключовi слова: диференцiальний, повiльно змiнний, ряди Фур'є.

Щёголев С. А. О приведении нелинейной дифференциальной системы второго порядка к одному специальному виду. Для нелинейной колебательной системы второго порядка построено преобразование, приводящее эту систему к близкой к системе с медленно меняющимися коэффициентами.
Ключевые слова: дифференциальный, медленно меняющийся, ряды Фурье.

Shchogolev S. A. On a reduction of nonlinear second order differential system to a some special kind. For nonlinear oscillating second-order differential system construct the transformation which reducing this system close to a system with slowly varying coefficients.
Key words: differential, slowly varying, Fourier series.

1. Introduction. One of the powerful methods of the study of nonlinear systems of differential equations is the method of integral manifolds [1,2]. Particularly important role it plays in the research of multi-frequency oscillations, in particular, in systems containing the slowly varying parameters [3]. An important object of study in the same time and are single-frequency system [4]. In the research of integral manifolds of nonlinear systems of differential equations is usually necessary to bring the system prior to some simpler form. The construction of the corresponding transformations in many cases leads in turn to auxiliary systems of differential equations that require special study. Therefore, the task of constructing these transformations can be regarded as independent. In this paper, a second order nonlinear system of differential equations, the coefficients of which are represented by an absolutely and uniformly convergent Fourier series with slowly varying parameters. For such a system, that is, systems with coefficients oscillating type, construct a transformation with coefficients of a similar structure, which reducing the system close to a system with slowly varying coefficients.

2. Basic notation and definitions.

Let $G = \{t, \varepsilon : \ t \in \mathbf{R}, \ \varepsilon \in [0, \varepsilon_0], \ \varepsilon_0 \in \mathbf{R}^+\}$.

Definition 1. We say, that a function $f(t, \varepsilon)$, in general a complex-valued, belongs to the class $S_m(\varepsilon_0)$, $m \in \mathbf{N} \cup \{0\}$, if $t, \varepsilon \in G$ and

1) $f(t, \varepsilon) \in C^m(G)$ with respect t,

2) $d^k f(t, \varepsilon)/dt^k = \varepsilon^k f_k^*(t, \varepsilon)$ $(0 \leq k \leq m)$,

$$\|f\|_m \stackrel{def}{=} \sum_{k=0}^{m} \sup_G |f_k^*(t, \varepsilon)|.$$

Definition 2. We say, that a function $f(t,\varepsilon,\theta(t,\varepsilon))$ belongs to the class $F^{\theta}_{m,l}(\varepsilon_0)$ $(m, l \in \mathbf{N}\cup\{0\})$, if this function can be represented as:

$$f(t,\varepsilon,\theta(t,\varepsilon)) = \sum_{n=-\infty}^{\infty} f_n(t,\varepsilon)\exp\left(in\theta(t,\varepsilon)\right),$$

and:

1) $f_n(t,\varepsilon) \in S_m(\varepsilon_0)$;

2)

$$\|f\|_{m,l} \overset{def}{=} \|f_0\|_m + \sum_{n=-\infty}^{\infty} |n|^l \|f_n\|_m < +\infty,$$

particular

$$\|f\|_{m,0} = \sum_{n=-\infty}^{\infty} \|f_n\|_m;$$

3) $\theta(t,\varepsilon)$ – the real function on G.

Definition 3. We say, that a function $f(t,\varepsilon,x)$ belongs to the class $\widetilde{S}^{x}_{m,q}(\varepsilon_0,d)$, if this function belongs to the class $S_m(\varepsilon_0)$ with respect t,ε and to the class $C^q[-d,d]$ with respect x, $|x|\le d$.

Definition 4. We say, that a function $f(t,\varepsilon,\theta(t,\varepsilon),x)$ belongs to the class $\widetilde{F}^{\theta,x}_{m,l,q}(\varepsilon_0,d)$, if this function belongs to the class $F^{\theta}_{m,l}(\varepsilon_0)$ with respect t,ε,θ and to the class $C^q[-d,d]$ with respect x, $|x|\le d$.

3. Statement of the Problem.

Consider the following system of differential equations:

$$\begin{aligned} \tfrac{dx}{dt} &= \mu X(t,\varepsilon,\theta,x) + \varepsilon a(t,\varepsilon,\theta,x), \\ \tfrac{d\theta}{dt} &= \omega(t,\varepsilon) + \mu\Theta(t,\varepsilon,\theta,x) + \varepsilon b(t,\varepsilon,\theta,x), \end{aligned} \tag{1}$$

where $x\in\mathbf{R}$, $|x|\le d$, $X,\ \Theta,\ \omega,\ a,\ b\in\mathbf{R}$, $X,\ \Theta \in \widetilde{F}^{\theta,x}_{m,l,q}(\varepsilon_0,d)$, $a,b\in\widetilde{F}^{\theta,x}_{m-1,l,q}(\varepsilon_0,d)$, $\omega\in S_m(\varepsilon_0)$, $\inf\limits_{G}\omega=\omega_0>0$; $\mu\in(0,\mu_0)$.

We study the question of the existence of the transformation

$$x = y + \mu u(t,\varepsilon,\varphi,y,\mu), \quad \theta = \varphi + \mu v(t,\varepsilon,\varphi,y,\mu), \tag{2}$$

where $|y|\le d_1 < d$, $u,v\in\widetilde{F}^{\varphi,y}_{m_1,l_1,q_1}(\varepsilon_0,d_1)$, which reducing the system (1) to the form:

$$\begin{aligned} \tfrac{dy}{dt} &= \mu Y(t,\varepsilon,y,\mu) + \varepsilon\widetilde{a}(t,\varepsilon,\varphi,y,\mu), \\ \tfrac{d\varphi}{dt} &= \omega(t,\varepsilon) + \mu\Phi(t,\varepsilon,y,\mu) + \varepsilon\widetilde{b}(t,\varepsilon,\varphi,y,\mu), \end{aligned} \tag{3}$$

where $Y,\Phi\in\widetilde{S}^{y}_{m,q_2}(\varepsilon_0,d_1)$, $\widetilde{a},\widetilde{b}\in\widetilde{F}^{\varphi,y}_{m-1,l_2,q_2}(\varepsilon_0,d_1)$. The numbers m_1,l_1,q_1,m_2,l_2,q_2 in some way dependent from m,l,q.

4. Principal Results.

Theorem. $\forall r\in\mathbf{N}$, $r<l$, $r<q$ $\exists\mu_r\in(0,\mu_0)$: $\forall\mu\in(0,\mu_r)$ *exist the transformation of kind*

$$x = y + \sum_{k=1}^{r} u_k(t,\varepsilon,\varphi,y)\mu^k, \quad \theta = \varphi + \sum_{k=1}^{r} v_k(t,\varepsilon,\varphi,y)\mu^k, \tag{4}$$

$|y| \le d_1 < d$, $u_k, v_k \in \widetilde{F}^{\varphi,y}_{m,l-k+1,q-k+1}(\varepsilon_0, d_1)$ $(k = \overline{1,r})$, *which reducing the system (1) to the form:*

$$\frac{dy}{dt} = \sum_{k=1}^{r} Y_k(t,\varepsilon,y)\mu^k + \mu^{r+1}\widetilde{Y}_r(t,\varepsilon,\varphi,y,\mu) + \varepsilon a_r(t,\varepsilon,\varphi,\mu),$$
$$\frac{d\varphi}{dt} = \omega(t,\varepsilon) + \sum_{k=1}^{r} \omega_k(t,\varepsilon,y)\mu^k + \mu^{r+1}\widetilde{\omega}_r(t,\varepsilon,\varphi,y,\mu) + \varepsilon b_r(t,\varepsilon,\varphi,\mu), \tag{5}$$

where $Y_k,\ \omega_k \in \widetilde{S}^{y}_{m,q-k+1}(\varepsilon_0, d_1)$, $\widetilde{Y}_r,\ \widetilde{\omega}_r \in \widetilde{F}^{\varphi,y}_{m,l-r,q-r}(\varepsilon_0, d_1)$, $a_r,\ b_r \in$ $\in \widetilde{F}^{\varphi,y}_{m-1,l-r,q-r}(\varepsilon_0, d_1)$.

Proof. We sustitute relations (4) in system (1) and require that the transformed system have the form (5). Then we obtain the following systems of the differential equations for u_k, v_k $(k = \overline{1,r})$:

$$Y_1(t,\varepsilon,y) + \omega(t,\varepsilon)\frac{\partial u_1}{\partial \varphi} = X(t,\varepsilon,\varphi,y),$$
$$\omega_1(t,\varepsilon,y) + \omega(t,\varepsilon)\frac{\partial v_1}{\partial \varphi} = \Theta(t,\varepsilon,\varphi,y), \tag{6}$$

$$Y_2(t,\varepsilon,y) + Y_1(t,\varepsilon,y)\frac{\partial u_1}{\partial y} + \omega_1(t,\varepsilon,y)\frac{\partial u_1}{\partial \varphi} + \omega(t,\varepsilon)\frac{\partial u_2}{\partial \varphi} =$$
$$= \frac{\partial X(t,\varepsilon,\varphi,y)}{\partial \varphi}\, v_1 + \frac{\partial X(t,\varepsilon,\varphi,y)}{\partial y}\, u_1,$$
$$\omega_2(t,\varepsilon,y) + Y_1(t,\varepsilon,y)\frac{\partial v_1}{\partial y} + \omega_1(t,\varepsilon,y)\frac{\partial v_1}{\partial \varphi} + \omega(t,\varepsilon)\frac{\partial v_2}{\partial \varphi} =$$
$$= \frac{\partial \Theta(t,\varepsilon,\varphi,y)}{\partial \varphi}\, v_1 + \frac{\partial \Theta(t,\varepsilon,\varphi,y)}{\partial y}\, u_1, \tag{7}$$

$$Y_j(t,\varepsilon,y) + \sum_{k=1}^{j-1} Y_{j-k}(t,\varepsilon,y)\frac{\partial u_k}{\partial y} + \sum_{k=1}^{j-1} \omega_{j-k}(t,\varepsilon,y)\frac{\partial u_k}{\partial \varphi} + \omega(t,\varepsilon)\frac{\partial u_j}{\partial \varphi} =$$
$$= \frac{\partial X(t,\varepsilon,\varphi,y)}{\partial \varphi}\, v_{j-1} + \frac{\partial X(t,\varepsilon,\varphi,y)}{\partial y}\, u_{j-1} + P_j(t,\varepsilon,\varphi,y,v_1,\dots,v_{j-2},u_1,\dots,u_{j-2}),$$
$$\omega_j(t,\varepsilon,y) + \sum_{k=1}^{j-1} Y_{j-k}(t,\varepsilon,y)\frac{\partial v_k}{\partial y} + \sum_{k=1}^{j-1} \omega_{j-k}(t,\varepsilon,y)\frac{\partial v_k}{\partial \varphi} + \omega(t,\varepsilon)\frac{\partial v_j}{\partial \varphi} =$$
$$= \frac{\partial \Theta(t,\varepsilon,\varphi,y)}{\partial \varphi}\, v_{j-1} + \frac{\partial \Theta(t,\varepsilon,\varphi,y)}{\partial y}\, u_{j-1} + Q_j(t,\varepsilon,\varphi,y,v_1,\dots,v_{j-2},u_1,\dots,u_{j-2}),$$
$$j = \overline{3,r}, \tag{8}$$

where P_j, Q_j – the polynomials with respect $v_1,\dots,v_{j-2},u_1,\dots,u_{j-2}$ whose coefficients dependent from derivatives up to the $(j-1)$-order with respect φ, y from functions X, Θ. The functions $\widetilde{Y}_r$, $\widetilde{\omega}_r$, a_r, b_r defined from the following systems of the linear algebraic equations:

$$\left(1 + \sum_{k=1}^{r} \frac{\partial u_k}{\partial y}\, \mu^k\right)\widetilde{Y}_r + \left(\sum_{k=1}^{r} \frac{\partial u_k}{\partial \varphi}\, \mu^k\right)\widetilde{\omega}_r =$$
$$= -\sum_{k=1}^{r} \frac{\partial u_k}{\partial y}\left(\sum_{s=r-k+1}^{r} Y_s\mu^{s-r+k-1}\right) - \sum_{k=1}^{r} \frac{\partial u_k}{\partial \varphi}\left(\sum_{s=r-k+1}^{r} \omega_s\mu^{s-r+k-1}\right) +$$
$$+\widetilde{P}_r(t,\varepsilon,\varphi,y,v_1,\dots,v_r,u_1,\dots,u_r),$$

$$\left(\sum_{k=1}^{r} \frac{\partial v_k}{\partial y}\ \mu^k\right) \widetilde{Y}_r + \left(1 + \sum_{k=1}^{r} \frac{\partial v_k}{\partial \varphi}\ \mu^k\right) \widetilde{\omega}_r =$$

$$= - \sum_{k=1}^{r} \frac{\partial v_k}{\partial y} \left(\sum_{s=r-k+1}^{r} Y_s \mu^{s-r+k-1}\right) - \sum_{k=1}^{r} \frac{\partial v_k}{\partial \varphi} \left(\sum_{s=r-k+1}^{r} \omega_s \mu^{s-r+k-1}\right) +$$

$$+ \widetilde{Q}_r(t, \varepsilon, \varphi, y, v_1, \ldots, v_r, u_1, \ldots, u_r), \tag{9}$$

$$\left(1 + \sum_{k=1}^{r} \frac{\partial u_k}{\partial y}\ \mu^k\right) a_r + \left(\sum_{k=1}^{r} \frac{\partial u_k}{\partial \varphi}\ \mu^k\right) b_r =$$

$$= -\frac{1}{\varepsilon} \left(\sum_{k=1}^{r} \frac{\partial u_k}{\partial t}\ \mu^k\right) + a\left(t, \varepsilon, \varphi + \sum_{k=1}^{r} v_k \mu^k, y + \sum_{k=1}^{r} u_k \mu^k\right),$$

$$\left(\sum_{k=1}^{r} \frac{\partial v_k}{\partial y}\ \mu^k\right) a_r + \left(1 + \sum_{k=1}^{r} \frac{\partial v_k}{\partial \varphi}\ \mu^k\right) b_r = \tag{10}$$

$$= -\frac{1}{\varepsilon} \left(\sum_{k=1}^{r} \frac{\partial v_k}{\partial t}\ \mu^k\right) + b\left(t, \varepsilon, \varphi + \sum_{k=1}^{r} v_k \mu^k, y + \sum_{k=1}^{r} u_k \mu^k\right),$$

$\widetilde{P}_r,\ \widetilde{Q}_r \in \widetilde{F}^{\varphi,y}_{m,l-r,q-r}(\varepsilon_0, d_1)$.

Consider the system (6). Expand the functions X, Θ in the Fourier series with respect φ:

$$X(t, \varepsilon, \varphi, y) = \sum_{n=-\infty}^{\infty} X_n(t, \varepsilon, y) \exp(in\varphi),\ \ \Theta(t, \varepsilon, \varphi, y) = \sum_{n=-\infty}^{\infty} \Theta_n(t, \varepsilon, y) \exp(in\varphi).$$

Solution of the system (6) found in the form of a Fourier series also:

$$u_1(t, \varepsilon, \varphi, y) = \sum_{n=-\infty}^{\infty} u_{1n}(t, \varepsilon, y) \exp(in\varphi),\ \ v_1(t, \varepsilon, \varphi, y) = \sum_{n=-\infty}^{\infty} v_{1n}(t, \varepsilon, y) \exp(in\varphi).$$

We substitute these expressions in the system (6), assuming:

$$Y_1(t, \varepsilon, y) = X_0(t, \varepsilon, y),\ \ \omega_1(t, \varepsilon, y) = \Theta_0(t, \varepsilon, y), \tag{11}$$

We obtain:

$$u_1(t, \varepsilon, \varphi, y) = \sum_{\substack{n=-\infty \\ (n \neq 0)}}^{\infty} \frac{X_n(t, \varepsilon, y)}{in\omega(t, \varepsilon)} \exp(in\varphi), \tag{12}$$

$$v_1(t, \varepsilon, \varphi, y) = \sum_{\substack{n=-\infty \\ (n \neq 0)}}^{\infty} \frac{\Theta_n(t, \varepsilon, y)}{in\omega(t, \varepsilon)} \exp(in\varphi). \tag{13}$$

Obviously, that $u_1, v_1 \in \widetilde{F}^{\varphi,y}_{m,l+1,q}(\varepsilon_0, d_1)$.

We denote:

$$g_1(t, \varepsilon, \varphi, y) = \frac{\partial X(t, \varepsilon, \varphi, y)}{\partial \varphi}\ v_1 + \frac{\partial X(t, \varepsilon, \varphi, y)}{\partial y}\ u_1 - Y_1(t, \varepsilon, y) \frac{\partial u_1}{\partial y} - \omega_1(t, \varepsilon, y) \frac{\partial u_1}{\partial \varphi},$$

$$h_1(t, \varepsilon, \varphi, y) = \frac{\partial \Theta(t, \varepsilon, \varphi, y)}{\partial \varphi}\ v_1 + \frac{\partial \Theta(t, \varepsilon, \varphi, y)}{\partial y}\ u_1 - Y_1(t, \varepsilon, y) \frac{\partial v_1}{\partial y} - \omega_1(t, \varepsilon, y) \frac{\partial v_1}{\partial \varphi}.$$

Then $g_1, h_1 \in \widetilde{F}^{\varphi,y}_{m,l-1,q-1}(\varepsilon_0, d_1)$. System (7) takes the form:

$$\begin{aligned} &Y_2(t,\varepsilon,y) + \omega(t,\varepsilon)\tfrac{\partial u_2}{\partial \varphi} = g_1(t,\varepsilon,\varphi,y),\\ &\omega_2(t,\varepsilon,y) + \omega(t,\varepsilon)\tfrac{\partial v_2}{\partial \varphi} = h_1(t,\varepsilon,\varphi,y). \end{aligned} \tag{14}$$

Expand the functions g_1, h_1 in the Fourier series with respect φ:

$$g_1(t,\varepsilon,\varphi,y) = \sum_{n=-\infty}^{\infty} g_{1n}(t,\varepsilon,y)\exp(in\varphi),$$

$$h_1(t,\varepsilon,\varphi,y) = \sum_{n=-\infty}^{\infty} h_{1n}(t,\varepsilon,y)\exp(in\varphi).$$

Similarly formulas (11), (12), (13) we obtain:

$$Y_2(t,\varepsilon,y) = g_{10}(t,\varepsilon,y),\ \omega_2(t,\varepsilon,y) = h_{10}(t,\varepsilon,y), \tag{15}$$

$$u_2(t,\varepsilon,\varphi,y) = \sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty} \frac{g_{1n}(t,\varepsilon,y)}{in\omega(t,\varepsilon)} \exp(in\varphi), \tag{16}$$

$$v_2(t,\varepsilon,\varphi,y) = \sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty} \frac{h_{1n}(t,\varepsilon,y)}{in\omega(t,\varepsilon)} \exp(in\varphi). \tag{17}$$

Obviously, that $u_2, v_2 \in \widetilde{F}^{\varphi,y}_{m,l,q-1}(\varepsilon_0, d_1)$.

Continuing the same way, in the j-s step we assume:

$$\begin{aligned} &g_{j-1}(t,\varepsilon,\varphi,y) = \tfrac{\partial X(t,\varepsilon,\varphi,y)}{\partial \varphi}\ v_{j-1} + \tfrac{\partial X(t,\varepsilon,\varphi,y)}{\partial y}\ u_{j-1} -\\ &- \textstyle\sum_{k=1}^{j-1} Y_{j-k}(t,\varepsilon,y)\tfrac{\partial u_k}{\partial y} - \sum_{k=1}^{j-1} \omega_{j-k}(t,\varepsilon,y)\tfrac{\partial u_k}{\partial \varphi} +\\ &+P_j(t,\varepsilon,\varphi,y,v_1,\dots,v_{j-2},u_1,\dots,u_{j-2}),\\ &h_{j-1}(t,\varepsilon,\varphi,y) = \tfrac{\partial \Theta(t,\varepsilon,\varphi,y)}{\partial \varphi}\ v_{j-1} + \tfrac{\partial \Theta(t,\varepsilon,\varphi,y)}{\partial y}\ u_{j-1} -\\ &- \textstyle\sum_{k=1}^{j-1} Y_{j-k}(t,\varepsilon,y)\tfrac{\partial v_k}{\partial y} - \sum_{k=1}^{j-1} \omega_{j-k}(t,\varepsilon,y)\tfrac{\partial v_k}{\partial \varphi} +\\ &+Q_j(t,\varepsilon,\varphi,y,v_1,\dots,v_{j-2},u_1,\dots,u_{j-2}), \end{aligned} \tag{18}$$

Functions g_{j-1}, h_{j-1} belongs to the class $\widetilde{F}^{\varphi,y}_{m,l-j+1,q-j+1}(\varepsilon_0, d_1)$.

System (8) takes the form:

$$\begin{aligned} &Y_j(t,\varepsilon,y) + \omega(t,\varepsilon)\tfrac{\partial u_j}{\partial \varphi} = g_{j-1}(t,\varepsilon,\varphi,y),\\ &\omega_j(t,\varepsilon,y) + \omega(t,\varepsilon)\tfrac{\partial v_j}{\partial \varphi} = h_{j-1}(t,\varepsilon,\varphi,y),\ j = \overline{3,r}. \end{aligned}$$

Expand the functions g_{j-1}, h_{j-1} in the Fourier series with respect φ:

$$g_{j-1}(t,\varepsilon,\varphi,y)=\sum_{n=-\infty}^{\infty} g_{j-1,n}(t,\varepsilon,y)\exp(in\varphi),$$

$$h_{j-1}(t,\varepsilon,\varphi,y)=\sum_{n=-\infty}^{\infty} h_{j-1,n}(t,\varepsilon,y)\exp(in\varphi).$$

Then

$$Y_j(t,\varepsilon,y)=g_{j-1,0}(t,\varepsilon,y),\ \omega_j(t,\varepsilon,y)=h_{j-1,0}(t,\varepsilon,y),\ j=\overline{3,r},$$

$$u_j(t,\varepsilon,\varphi,y)=\sum_{\substack{n=-\infty\\(n\neq 0)}}^{\infty}\frac{g_{j-1,n}(t,\varepsilon,y)}{in\omega(t,\varepsilon)}\exp(in\varphi),\ j=\overline{3,r},$$

$$v_j(t,\varepsilon,\varphi,y)=\sum_{\substack{n=-\infty\\(n\neq 0)}}^{\infty}\frac{h_{j-1,n}(t,\varepsilon,y)}{in\omega(t,\varepsilon)}\exp(in\varphi), j=\overline{3,r}.$$

Obviously $u_j,\ v_j\in\widetilde{F}^{\varphi,y}_{m,l-j+2,q-j+1}(\varepsilon_0,d_1)$, $Y_j,\ \omega_j\in\widetilde{S}^{y}_{m,q-j+1}(\varepsilon_0,d_1)$ $(j=\overline{3,r})$.

Thus, the functions

$$u(t,\varepsilon,\varphi,y,\mu)=\sum_{k=1}^{r}u_k(t,\varepsilon,\varphi,y)\mu^{k-1},$$

$$v(t,\varepsilon,\varphi,y,\mu)=\sum_{k=1}^{r}v_k(t,\varepsilon,\varphi,y)\mu^{k-1}$$

belongs to the class $\widetilde{F}^{\varphi,y}_{m,l-r+2,q-r+1}(\varepsilon_0,d_1)$.

From correlations (9), (10) follows, that for sufficiently small μ: $\widetilde{Y}_r,\ \widetilde{\omega}_r\in$ $\in\widetilde{F}^{\varphi,y}_{m,l-r,q-r}(\varepsilon_0,d_1)$, $a_r,\ b_r\in\widetilde{F}^{\varphi,y}_{m-1,l-r,q-r}(\varepsilon_0,d_1)$.

Theorem are proved.

Conclusions. Thus, for the system (1) the sufficient conditions of the existence of the transformation, which reducing this system close to a system with slowly varying coefficients and the algorithm for constructing this transformation are obtained. This result can be used for the research of integral manifolds of the system (1), which represented by an absolutely and uniformly convergent Fourier series with slowly varying parameters.

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