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# Forced Vibrations of a Boxed Shell of Square Cross-Section

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**Abstract**—We construct the solution of the problem on the steady-state vibrations of a finite boxed shell of square cross-section with symmetry conditions at the shell ends. We present the dispersion curves, find the natural frequencies, and study the stress distribution in the shell. We obtain a simple formula for the approximate analysis of the shell in the case of low-frequency vibrations on the basis of the expansion of the solution in two small parameters and on the Lagrange interpolation formula.

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Key words: *forced vibrations, boxed shell, asymptotics, stressed state.*

The harmonic vibrations of a half-infinite boxed shell of rectangular profile were considered in [1], where homogeneous solutions were constructed. In [2], the dispersion equations were obtained for normal waves propagating in an unbounded boxed shell of angled and square cross-section. Note that no natural frequencies were found and no computations were performed in the cited papers. It is these problems that are studied in the present paper.

Consider the steady-state forced vibrations of a boxed shell (Fig. 1) that consists of thin plates of thickness $\tilde{h}$, width $2\tilde{a}$, and length $\tilde{l}$ (in dimensional quantities). At the shell ends (for $\tilde{y} = 0, \tilde{l}$), we pose the symmetry conditions. The same transverse load symmetric with respect to the shell edges is applied to each of the plates. Since the shell has four planes of symmetry, it suffices to consider the mathematical statement of the problem for the rectangular plate shown in Fig. 2.

In dimensionless form, the boundary value problem describing the simultaneous plane-bending stress state of the plates of the boxed shell consists of the differential equation

$$D\Delta^2 w(x,y) - \omega^2\varepsilon^{-2} w(x,y) = q(x,y), \quad 0 < x < 1, \quad 0 < y < l, \tag{1}$$

of bending vibrations of a thin plate, the Lamé equations

$$\begin{aligned} &\frac{1}{G}\Delta u(x,y) + \frac{2}{1-\mu}\frac{\partial\theta(x,y)}{\partial x} + \omega^2 u(x,y) = 0, \\ &\frac{1}{G}\Delta v(x,y) + \frac{2}{1-\mu}\frac{\partial\theta(x,y)}{\partial y} + \omega^2 v(x,y) = 0, \quad 0 < x < 1, \quad 0 < y < l, \end{aligned} \tag{2}$$

describing the plane stress state of the plate, the boundary conditions

$$\left.\frac{\partial w}{\partial x}\right|_{x=0} = 0, \quad V_x\big|_{x=0} = 0, \quad u\big|_{x=0} = 0, \quad \tau_{xy}\big|_{x=0} = 0 \tag{3}$$

taking into account the symmetry of the load with respect to the $y$-axis, the boundary conditions

$$\left.\frac{\partial w}{\partial x}\right|_{x=1} = 0, \quad \tau_{xy}\big|_{x=1} = 0, \quad w\big|_{x=1} = -\varepsilon^2 u\big|_{x=1}, \quad V_x\big|_{x=1} = \sigma_x\big|_{x=1} \tag{4}$$

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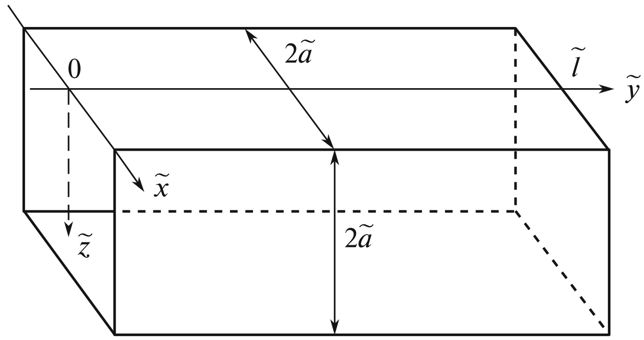


**Fig. 1.**

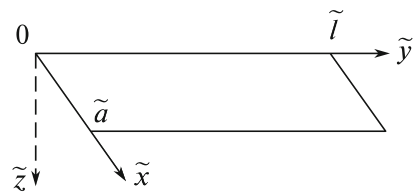


**Fig. 2.**

describing the rigid connection of the plates with the symmetry with respect to the shell edges taken into account (see [3]), and the boundary conditions

$$\left.\frac{\partial w}{\partial y}\right|_{y=0,l} = 0, \quad V_y\big|_{y=0,l} = 0, \quad v\big|_{y=0,l} = 0, \quad \tau_{xy}\big|_{y=0,l} = 0 \tag{5}$$

of symmetry at the shell ends.

The factor $e^{-i\omega t}$ is omitted for all the variables. The dimensional variables (in what follows, they are marked by tildes) are related to the dimensionless quantities as follows: $\tilde{x} = \tilde{a}x$, $\tilde{y} = \tilde{a}y$, $\tilde{l} = \tilde{a}l$, $\tilde{h} = \tilde{a}\varepsilon$, $\tilde{D} = \tilde{E}\tilde{h}^3 D$, $\tilde{G} = \tilde{E}C$, $\tilde{q} = \tilde{E}q$, $\tilde{w} = \tilde{a}\varepsilon^{-3}w$, $\tilde{u} = \tilde{a}\varepsilon^{-1}u$, $\tilde{v} = \tilde{a}\varepsilon^{-1}v$, $\tilde{V}_{\tilde{x}} = \tilde{E}\tilde{a}V_x$, $\tilde{\sigma}_{\tilde{x}} = \tilde{E}\sigma_x$, $\tilde{\tau}_{\tilde{x}\tilde{y}} = \tilde{E}\tau_{xy}$, $\tilde{\omega} = \omega\tilde{T}^{-1}$, $\tilde{T} = \tilde{a}/\tilde{c}$, $\tilde{c} = \sqrt{\tilde{E}/\tilde{\rho}}$, and $\tilde{t} = t\tilde{T}$; $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ are the displacements of the plate points in the direction of the $\tilde{x}$-, $\tilde{y}$-, and $\tilde{z}$-axes, respectively;

$$\tilde{V}_{\tilde{x}} = -\tilde{D}\left[\frac{\partial^3\tilde{w}}{\partial\tilde{x}^3} + (2-\mu)\frac{\partial^3\tilde{w}}{\partial\tilde{x}\partial\tilde{y}^2}\right], \quad \tilde{D} = \frac{\tilde{E}\tilde{h}^3}{12(1-\mu^2)}, \quad \tilde{\sigma}_{\tilde{x}} = \tilde{F}\left(\frac{\partial\tilde{u}}{\partial\tilde{x}} + \mu\frac{\partial\tilde{v}}{\partial\tilde{y}}\right), \quad \tilde{\tau}_{\tilde{x}\tilde{y}} = \tilde{G}\left(\frac{\partial\tilde{u}}{\partial\tilde{y}} + \frac{\partial\tilde{v}}{\partial\tilde{x}}\right),$$

where $\tilde{V}_{\tilde{x}}$, $\tilde{\sigma}_{\tilde{x}}$, and $\tilde{\tau}_{\tilde{x}\tilde{y}}$ are the generalized transverse force and the normal and tangential stresses over the cross-section with normal $\tilde{x}$; $\tilde{D}$ is the cylindrical rigidity of the plates; $\tilde{h}$ is the plate thickness; $2\tilde{a}$ is the plate width; $\tilde{l}$ is the plate length; $\tilde{\rho}$ is the plate material density; $\tilde{E}$ is the Young modulus; $\mu$ is the Poisson ratio; $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplace operator; $\tilde{G} = \tilde{E}/[2(1+\mu)]$ is the shear modulus; $\theta = \partial u/\partial x + \partial v/\partial y$ and $\tilde{F} = \tilde{E}/(1-\mu^2)$.

Note that the boundary conditions (5) correspond to the problem on steady-state vibrations of an infinite boxed shell under a load $l$-periodic in the variable $y$. The problem with the Navier conditions

$$w\big|_{y=0,l} = 0, \quad M_y\big|_{y=0,l} = 0, \quad u\big|_{y=0,l} = 0, \quad \sigma_y\big|_{y=0,l} = 0$$

at the shell ends can be obtained from the solution of problem (1)–(5) with periods $l$ and $2l$.

By expressing $u$ and $v$ via the potentials $u = \partial\varphi/\partial x + \partial\varphi/\partial y$ and $v = \partial\varphi/\partial y - \partial\varphi/\partial x$ and by applying the finite Fourier transform in the variable $y$ on the interval $[0, l]$ with parameter $\alpha = \pi m/l$ to