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PROBLEM OF A RANDOMLY ORIENTED CRACK IN A BOX-SHAPED SHELL

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This paper deals with the stress state of a box-shaped shell formed by two semi-infinite plates joined at a right angle. The plates are homogeneous but have different thicknesses. The shell is weakened by a finite rectilinear crack of unit length which reaches one edge of the shell. The orientation of the crack and the load on its edges are arbitrarily chosen. The problem is solved with the assumption that the thickness of the plates is small compared to the length of the crack, which allows an asymptotic formulation of the problem. The problem is reduced to a special type of Riemannian vector problem in which the stress-intensity factor allows matrix factorization in accordance with Khrapkov's scheme. The asymptotes of the resulting solution and the stress-intensity factor are examined in relation to the thickness of the shell and the angle formed by the crack and the edge of the shell.

1. We are examining the stress state of a box-shaped shell formed by two semi-infinite plates butt-joined at a right angle. The plates are uniform but have different thicknesses. The shell is weakened by a straight finite crack of unit length which extends to the edge of the shell. The crack is randomly oriented. Normal and shear loads act along the edges of the crack (in the middle plane of the plate containing the crack). The problem is solved with the assumption that the thicknesses of the plates are small compared to the length of the crack, which makes it possible to examine the problem in an asymptotic formulation [4].

The initial problem is reduced [4] to determination of the combined stress state that in-plane bending creates in a hypothetical plate with intersecting defects. The crack and the edge it reaches play the role of these defects. By a defect here, we mean a line the crossing of which results in discontinuity of the stresses or displacements of points of the shell. This approach has two advantages. First of all, it reduces the number of equations that have to be solved and the number of boundary conditions that must be satisfied. Secondly, a method for solving problems by this approach has already been developed [3]. The presence of a small parameter allows the initial problem to be reduced to successively solved problems on the bending of a plate and the problem of the stress-strain state of the plate.

In the case of a load that is symmetric relative to the crack, the initial problem reduces to the solution of equations of the two-dimensional theory of elasticity [2] which satisfy the joining conditions

$$< u > = 0 , \quad < \tau > = K_\theta \, \tau_{r\theta} \,|_{\theta+0} , \quad \sigma_\theta^1 = \sigma_\theta^2 = 0,$$

$$(\theta = 0 , \theta = \pi) , \quad 0 < r < \infty \tag{1.1}$$

and the conditions on the crack

$$\sigma_\theta = f_1 (r) , \quad \tau_{r\theta} = f_2 (r) \text{ at } \theta = \alpha , \quad 0 < r < 1 . \tag{1.2}$$

Here, u and v are the radial and tangential displacements of points of the shell; σ_r, σ_θ, and $\tau_{r\theta}$ are the normal and shear stresses in the two-dimensional polar coordinate system of the "hypothetical" compound plate formed by the merging of two local cylindrical systems in the flanges of the shell.

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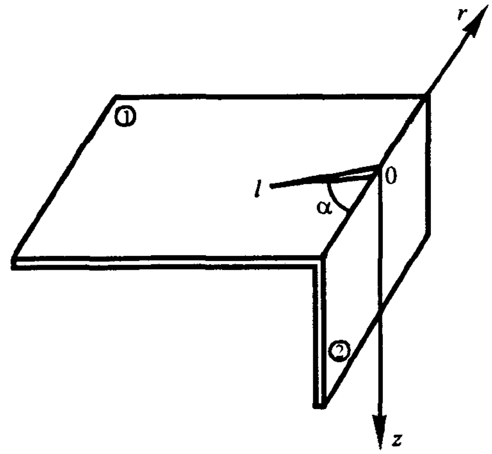

Fig. 1

In these coordinates, $u^{(j)}$, $v^{(j)}$ are the radial and tangential displacements of points of the jth plate; $\sigma_r^{(j)}$, $\sigma_\theta^{(j)}$, $\tau_{r\theta}^{(j)}$ are the corresponding stresses. These quantities are related to the solution of problem (1.1)–(1.2). Here, $j = 1$ corresponds to the case $\theta > 0$, while $j = 2$ corresponds to the case $\theta < 0$ (Fig. 1). Accordingly, the parameter K_θ

$$K_0 = 1 - h_1/h_2 \text{ at } \theta = 0,$$

$$K_\pi = 1 - h_2/h_1 \text{ at } \theta = \pi,$$

where h_1, h_2 are the thicknesses of the plates which form the shell.

We are using $<A> : = A\,|_{\theta - 0} - A\,|_{\theta + 0}$ to signify a jump discontinuity of the function $A(\gamma)$ at $\gamma = \theta$.

We must also require that the components of all of the stresses satisfy conditions of regularity

$$(\sigma_\theta, \sigma_r, \tau_{r\theta}) = 0\,(r^\lambda) \text{ at } r \to 0\ (\lambda > -1).$$

We designate

$$<v(r, \theta)>|_{\theta=\alpha} = \chi(r), \quad 0 < r < 1;$$

$$<u(r, \theta)>|_{\theta=\alpha} = \mu(r), \quad 0 < r < 1, \tag{1.3}$$

where

$$\chi(r) \equiv 0, \ \mu(r) \equiv 0 \text{ at } r > 1.$$

Using the generalized method of integral transforms [2] and introducing the Mellin transformation

$$M[A(r)] : = \int_0^\infty A(r)\, r^p\, dr, p \in \Lambda,$$

where Λ is a contour lying in the region $\{-1 < \operatorname{Re} p < 0, -\infty < \operatorname{Im} p < \infty\}$, we examine the Mellin transforms

$$[\sigma_{\theta p}, \sigma_{rp}, u_p, v_p, \tau_{r\theta p}] = M\left[\sigma_\theta, \sigma_r, u, v r^{-1}, \tau_{r\theta} r^{-1}\right]. \tag{1.4}$$