Induced maps and geodesic curves of Riemannian spaces of the second approximation

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For two Riemannian spaces $V_n(x,g)$ and $\overline{V}_n(x,\overline{g})$, where $\overline{V}_n$ admits a nontrivial geodesic mapping onto V_n ([1]) invariantly connected with them spaces of the second order $\widetilde{\overline{V}}{}^2_n(y,\widetilde{\overline{g}})$ and $\widetilde{V}^2_n(y,\widetilde{g})$ are constructed ([2]):

$$\widetilde{g}_{ij}(y) = \mathrm{g}_{ij} + \frac{1}{3}\mathrm{R}_{\cdot i\alpha\beta j}y^\alpha y^\beta, \tag{1}$$

$$\widetilde{\overline{g}}_{ij}(y) = \overline{\mathrm{g}}_{ij} + \frac{1}{3}\overline{\mathrm{R}}_{\cdot i\alpha\beta j}y^\alpha y^\beta, \tag{2}$$

where $\mathrm{g}_{ij} = g_{ij}(M_0)$, $\mathrm{R}_{\cdot i\alpha\beta j} = R_{i\alpha\beta j}(M_0)$.

We investigate the specificity of the map $\widetilde{\gamma}$ of the space $\widetilde{\overline{V}}{}^2_n$ onto the space $\widetilde{V}^2_n$, induced by a geodesic mapping γ of the initial spaces. We will find the deformation tensor of the mapping $\widetilde{\gamma}$ in the explicit form

$$\widetilde{P}^h_{ij} = \widetilde{\overline{\Gamma}}{}^h_{ij} - \widetilde{\Gamma}^h_{ij}. \tag{3}$$

We prove the absolute and uniform convergence of the obtained series.

By requiring that the map $\widetilde{\gamma}$ has been geodesic, we see that $\widetilde{\gamma}$ is affine. The following theorem is valid.

Theorem. *A nontrivial geodesic mapping γ of a Riemannian space of nonzero constant curvature $\overline{V}_n$ onto a space V_n induces an almost geodesic mappings of the third type $\prod_3$ of a space of the second order $\widetilde{\overline{V}}{}^2_n$ onto a space $\widetilde{V}^2_n$.*

A system of ordinary differential equations

$$\frac{d^2y^h}{ds^2} + \widetilde{\Gamma}^h_{\alpha\beta}\frac{dy^\alpha}{ds}\frac{dy^\beta}{ds} = 0, \qquad (4)$$

determining the geodesic curves in $\widetilde{V}^2_n$ is investigated.

1. N. S. Sinyukov. *Geodesic mappings of Riemannian spaces*. M.: Nauka, (1979), 225p. (in Russian)
2. S. M. Pokas'. *Lie groups of motions in a Riemannian space of the second approximation*, Proc. Penza State Pedagogical University named after V.G. Belinsky, Ser. phys. and math. sciences, no. 26, (2011), P. 173–183. (in Russian)