RELATIVISTIC EFFECTS IN A SYSTEM OF GRAVITATIONALLY INTERACTING NON-DUST-LIKE PARTICLES

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ABSTRACT. The two-body problem in the Kaluza–Klein model with toroidal topology of additional spatial dimensions is investigated. The characteristic property of the considering gravitational field sources is the existence of tension into the compact subspace, which is necessary to achieve the agreement with experimental restrictions on the PPN-parameter γ. It is shown that the two-body problem does not possess the essential symmetry and is inconsistent in the abovementioned model.

Key words: Kaluza–Klein models: latent solitons, two-body problem.

It is known that in the $(\mathcal{D} = D + 1)$-dimensional Kaluza–Klein (KK) models with toroidal compactification of extra spatial dimensions (ESDs) a dust-like matter source generates the metrics characterized by the PPN-parameter γ of the form $\gamma = 1/(D - 2)$. This result does not depend on size of ESDs. At the same time there exists a strict experimental restriction on the value of γ, following from the Shapiro time-delay experiment using the Cassini spacecraft: $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$. Obviously, the dust-like source can provide $\gamma = 1$ only when the total number of spatial dimensions $D = 3$, i.e. in general relativity. To satisfy that restriction in KK models with considering type of compactification, we can introduce a class of matter sources, which are characterized by tension in the compact space. The static spherically symmetric metric generated by the solitary source of that class is called a *latent soliton* (LS), hence such class of sources may be called *LS-sources*. In particular, black strings are LS-sources, corresponding LS are black branes. In the present work we intended to clarify how noncontradictory from physical point of view are the models with LS-sources. We investigate a system of two LS-sources with energy-momentum tensor (EMT) of the form:

$$T^{M\tilde{N}} = \tilde{\rho} c^2 u^M u^{\tilde{N}},\ T^{\bar{\mu}\bar{\nu}} = -\omega_{(\bar{\mu}-3)} \tilde{\rho} c^2 g^{\bar{\mu}\bar{\nu}} \qquad (1a)$$

$$\tilde{\rho} \equiv \sum_{p=1}^{2} \left[(-1)^D g\right]^{-1/2} \delta(\mathbf{x} - \mathbf{x}_p) m_p \frac{ds}{dx^0}, \qquad (1b)$$

where $M = 0, \dots, 4$; $\tilde{N} = 0, \dots, D$; $\bar{\mu}, \bar{\nu} = 4, \dots, D$. $\omega_{(\bar{\mu}-3)}$ plays a role of the parameter of state into the compact subspace (e.g. $\omega_{(\bar{\mu}-3)} = -1/2$, $\forall \bar{\mu}$ for black strings). Using multidimensional Einstein equations we can find approximate (up to the $1/c^3$ and $1/c^4$ terms) expressions for the components of metric tensor, corresponding to the system. From these expressions it follows, that in the investigating model the PPN-parameter γ is given by the expression $\gamma = (1 - \Sigma)/(D - 2 + \Sigma)$, where $\Sigma \equiv \sum_{\forall \bar{\mu}} \omega_{(\bar{\mu}-3)}$. From experimental restrictions on γ it follows, that:

$$\gamma = 1 \quad \Rightarrow \quad \Sigma = (3 - D)/2. \qquad (3)$$

Also we construct the approximate expression for the Lagrangian of the system up to the $1/c^2$ terms. Using the relation between the one-particle and the two-particle Lagrangians: $\partial L_i^{(1)}/\partial \mathbf{r} = \partial \mathcal{L}^{(2)}/\partial \mathbf{r}_i$, $i = 1, 2$, where $L_i^{(1)} = -m_i c\, ds_i/dt$, we obtain two different two-particle Lagrangians $\mathcal{L}_i^{(2)}$. The symmetry of the two-body problem requires that $\mathcal{L}_1^{(2)} = \mathcal{L}_2^{(2)} = \mathcal{L}^{(2)}$. This equality involves the following one:

$$\Sigma = 0. \qquad (4)$$

It's obvious, that (3) and (4) are consistent only in case $D = 3$. Hence, the two-body problem could not be formulated noncontradictory in KK models with toroidal topology of ESDs.

References

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