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ECONOMIC INFORMATION PROCESSING SYSTEMS

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The textbook is written in accordance with the program of the course " Economic information processing systems ", which is read to junior specialists and bachelors in the specialty "Economics". The manual will be useful to identify and critically assess the state and trends of economic information processing systems and apply them to the formation of new models of economic systems and processes.

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CONTENT

INTRODUCTION

At present, Ukrainian enterprises, following the entire civilized world, have entered a period of not just a market economy, but "informational" or "digital" economy of a post-industrial society, when Information is considered as one of the main resources for the development of new economic relations.

The only way to survive today is to be able to *receive reliable information in sufficient amount in real time, process and use this information for making management decisions meaningfully and quickly. Until now, companies could experience a lack of information, since the search, collection and processing of information were lengthy and time-consuming processes. The new century creates other problems – it immerses us in an ocean of information and information technology and makes us learn to swim in this ocean, choosing only the most important and useful information from this ocean, as well as managing large flows of information from such an ocean. Effective technical means for the development of this ocean are computer information systems and "digital" technologies, which are applied to* ***economic information*** *which accompanies and reflects the processes of production, distribution, exchange and consumption, material goods and services* and constitutes the subject of our presentation. Since the rapid development of computer and information technologies have made significant changes in decision-making related to economic activity.

Economic information is a central component of the management of any economic object. ***Economic information*** is *information on the results of the business entity's activities,* is used by various categories of personnel for making managerial decisions: managers, economists, financiers, production organizers, etc. Economic information (EI) is also necessary for external users – third-party organizations and enterprises, tax authorities, authorities, investors, etc. Many management decisions depend on EI, and it has the most significant impact on the efficiency of automation of equalization.

A specificity of EI is its obligatoriness for all legal entities in the country (for societies and citizens conducting entrepreneurial activities without forming a legal entity).

The provided EI must satisfy the following requirements:

• the correct methodological basis for constructing planned and existing indicators;

• coverage and effectiveness of the organization of all business processes and operations;

• reliability, completeness, accuracy of data, efficiency of operations.

Since the volume of EI is quite large, especially in the current globalization of economic relations, for the effective management of the enterprise this leads to the need for automated processing of EI and the ways of its transmission and presentation, i.e. implementation of EI processing systems based on digital information technologies and the Internet. The enterprise's use of digital information technologies and the Internet in the EI processing system allows it:

- increase the degree of validity of decisions made through the rapid collection, transmission and processing of EI;

- ensure the timeliness of decision-making on managing the organization in a market economy;

- to achieve the growth of management efficiency due to the timely presentation of the necessary EI to managers of all levels of management from a single information fund;

- coordinate management decisions;

- due to the awareness of management personnel about the current state of the economic object, to ensure the growth of labor productivity, reduction of non-production losses, etc.

All of the above allows us to conclude that today the role of automated systems for processing EI based on digital information technologies and the Internet has significantly increased. Their creation leads to more coordinated decision-making processes in the formation of strategic economic competitiveness, both of individual enterprises and the country.

THEME 1. **MODERN SCIENTIFIC AND TECHNICAL PRESENTATION OF INFORMATION**

1.1. The modern generalized idea of information and its purpose.

Obtaining information from the outside world, its conservation, analysis and reproduction are one of the main functions of man, other living organisms and automatic information technology systems. Being accumulated and processed from certain positions, the information gives new information, leads to new knowledge about the world, which allows a person to carry out targeted activities to manage and transform the world around him to meet his needs. From this point of view, for information, the following clarification of this term can be given:

Information **-** *fundamental scientific concept, which is the main means for the consumer of information to achieve long-term and significant competitive advantages, ensuring the development of the right decisions in management*.

The progress of mankind inevitably entails an increase in the total amount of information that it has, and this volume is growing much faster than the world's population and its material needs. Information is contained in human speech, book texts, magazines, newspapers, radio and television messages, instrument readings, etc. A person perceives information through the senses. It stores and processes it with the help of the brain and central nervous system.

A feature of the concept of “information” is its complexity and universality, it is used in all areas of human activity without exception: in philosophy, in biology, medicine, in sociology, art, in technology and economics, and, of course, in everyday life. Initially, the meaning of the word "information" was interpreted as something inherent only in human consciousness and communication - knowledge, information, news. Then the meaning of the word began to expand and generalize. Thus, from the standpoint of the materialist theory of knowledge, one of the universal properties of matter (along with motion, space, time, etc.) was recognized as reflection, which consists in the ability to adequately display other real objects with one real object, and the fact that one object is reflected in another means the presence in it of information about the reflected object. Hence, as soon as the states of one object are in accordance with the states of another object (for example, the correspondence between the position of the arrow of a voltmeter and the voltage at its terminals or the correspondence between our sensation and reality), this means that one object reflects another, i.e. contains information about friend. Any physical (mechanical) process occurs only because any physical process is a process of exchange, interaction of material objects, in which each participant in the physical process receives information about the other participant in the process through physical influences (forces, impulses, waves, etc.) and changes his the parameters are exactly in accordance with the given forces, their configuration. In this way, according to modern concepts, information is one of the initial categories of the universe along with matter and energy, and these categories are closely interrelated. Without information, life in any form cannot exist, and any information systems created by man cannot function, but also the physical world in general. Due to the complexity and universality of the very concept of information, a unified definition of the term "information" has not been adopted at present. From the point of view of various fields of knowledge, this concept is described by its specific set of features. *Due to the complexity and universality of the very concept of information, a unified definition of the term "information" has not been adopted at present. From the point of view of various fields of knowledge, this concept is described by its specific set of features. Due to the complexity and universality of the very concept of information, a unified definition of the term "information" has not been adopted at present. From the point of view of various fields of knowledge, this concept is described by its specific set of features.*

To date, the properties of information have been well studied in a wide variety of disciplines. Physics, for example, deals with the properties of signals that carry information. As an applied discipline, there is an information theory that deals with the informational content of signals (messages). Information theory is closely related to physics and mathematics; it uses the methodological apparatus of radio engineering and probability theory. Genetics is studying the transmission of hereditary information in wildlife. This list can be continued, but there is still no fundamental science that studies the nature of information. Therefore, there is no rigorous scientific definition of information that is about one of the problematic issues of modern science. Here are a few examples in which attempts were made to give a generalized definition revealing the meaning of the term "information":

- ***information*** *– a measure of reducing a person's lack of awareness of his environment;*
- ***information*** *– documented or publicly announced information and phenomena that occur in society, the state and the environment;*
- ***information*** *– a collection of information, knowledge and messages about the object, phenomena and processes.*

From the first two definitions it follows that information is inherent only to man, that at the present stage of technological progress this is not true, and from the last definition it is not clear to whom and for what purposes the information is intended, and such concepts as information, knowledge and messages also need to be clarified.

However, the technical and legal support of the electronic regime a signature, the use of cryptography methods in civil workflow, the use of the Internet in e-commerce, data security and many other social processes and phenomena require a strict and consistent definition of information. It requires economic and legal disciplines.

Let us now consider the formation of the scientific and technical concept of the term “information” at the present stage.

1.2. Scientific and technical definition of information.

Firstly, the concept of "information" implies the presence of two objects - the source and consumer of information. Then the information is transmitted from the source to the consumer in a material and energy form, called a ***message*** and containing information of various nature intended for the recipient about the reality surrounding him, about what actions he should take, etc.

According to such representations, for the term "information" the following definition can be given, which is widely used in computer science:

Information *– this is information (messages) about the object, phenomena, processes, etc. transmitted by the source to the recipient (consumer).*

The set of technical means used to transmit information from a source to a recipient is called a communication system, and the set of technical devices or the physical environment that ensure the transmission of a message from a transmitter to a receiver is a ***communication channel***. Schematically, the process of transmitting information by a communication system is shown in Fig. 1.

The encoder is equipped with a method for storing information in the form of data.

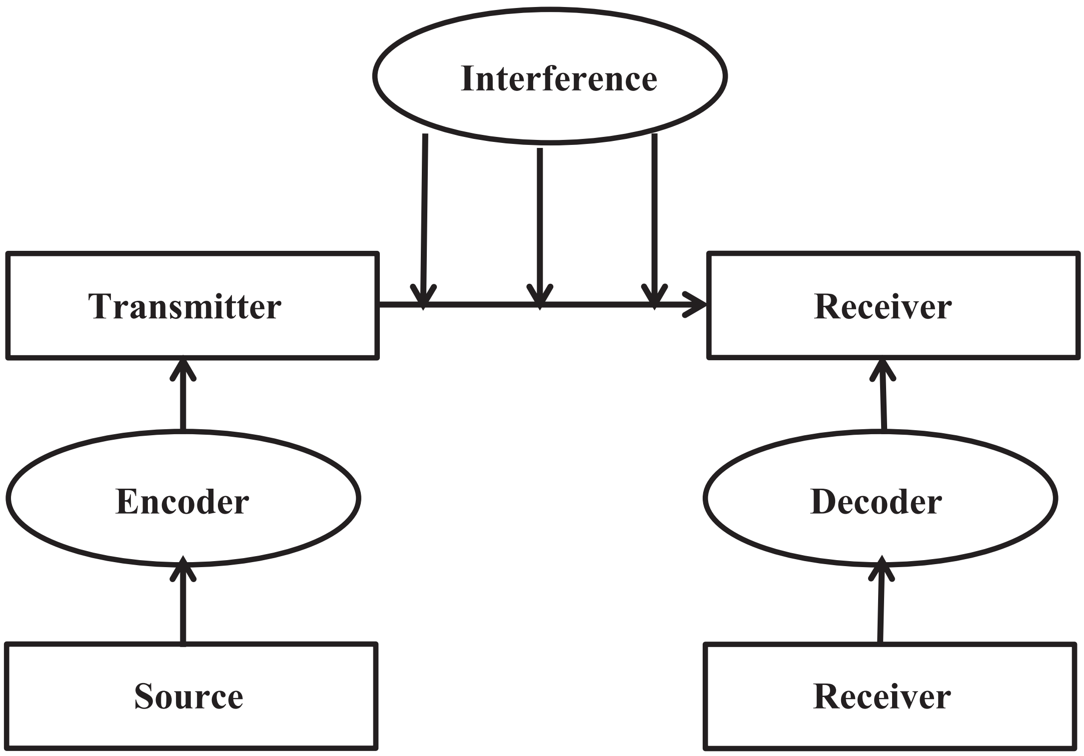


Fig. 1. The process of transmitting information

Data *– the result of recording (recording), displaying information on any material medium, that is, the presentation of information registered on the* *medium,* regardless of whether this information has reached any recipient and whether they are of interest to him.

Information is recorded in the form of data on a material medium (paper, magnetic tape, film, etc.) by changing its certain properties (color, relief, shape, chemical composition, etc.).Data resulting from the fixation of some information on a material medium is stored for a long (unlimited) time and is used to transmit information over time.

The transmitter converts this data into an information signal, which is a message designed to transmit it over the communication channel to the receiver.

Information signal *– a physical process, the change of parameters of which in time carries information to the recipient.*

A physical process can be a sound wave, an electric current, an electromagnetic wave, etc., while, as soon as the radiation of the signal stops, the flow of information also stops, that is, the transmission and reproduction of information, in contrast to data, is a dynamic object, and occur in real time.

The receiver converts the received signal over the communication channel again into data for which the decoding device has a method capable of reproducing the source information suitable for use by its recipient (bio-organism or man-made machine).

It should also be noted that the message can be transmitted over the communication channel and in the form of data, for example by mail, or by courier. Therefore, messages can be presented both in the form of information signals and in the form of data.

According to the above communication system diagram (Fig. 1), information is like, The information transmitted by the source to the consumer is a dynamic object that does not exist in nature by itself, but arises only in the presence of a consumer and is formed during the interaction of data and methods (in a decoding device) that ensure the reproduction of information for the consumer. Information process there is exactly as long as the interaction of data and methods lasts, and the rest of the time is in the form of a message: data or signal.

Based on the foregoing, we come to the following scientific and technical definition:

Information *is a product of the interaction of data and methods, considered in the context of this interaction.*

This definition does not say anything about the form in which the data is presented, it can be absolutely any. If the data is graphic, and the interaction method is observation, then *visual information* is generated. If the data is text or speech, and the method of their consumption is reading or listening, *text or audio information* is generated.

1.3. The concept context method.

In our scientific and technical definition of information the explanation *"...considered in the context of this interaction"* is important.

In everyday life, we often believe that books, newspapers, magazines, radio and television are sources of information. If you strictly approach from the standpoint of computer science, then this is not entirely true. All these are sources of messages, either data (books, newspapers, magazines), or information signals (radio and television). *Whether data and information signals will become a source of information or not, and if they become, which one, depends on which method will be applied to their consumption and in what conditions it will happen,* i.e., everything is determined by the consumption process in the presence of the consumer.

For example, already an answer to a simple question: "How much information is available in a computer science textbook for grades 10-11?" The student will immediately say: "For me, there is 0 information in it!" – and will be right! He knows him for a long time, and will not find *anything new* for himself in such a textbook.

One more example. During the Second World War, the American admirals faced the problem of how to organize negotiations between the courts, but at the same time ensure a high degree of secrecy? The Americans were aware of the success of the Japanese in decoding. The way out was found! A Cherokee Indian was planted on each of the ships: only a few hundred remained in the United States. So the highest level of secrecy was ensured! The Indians spoke among themselves in their "Cherokee" language, but the Japanese did not understand them! Of course, the Indians then translated the leadership of the ships and headquarters what they heard from their fellow tribesmen into English - but there was simply no one to make such a translation to the Japanese! Thus, for the Americans, this Native American tale that filled the radio of the Pacific Ocean was “full of information”,

Just two examples – but they reveal an extremely important feature of the information. It turns out that an important place, in this case, also has the issue of the *readiness* of the consumer of information to perceive it.

Nevertheless, we are still used to talking not about the transmission of messages, that is, data and information signals, but about the transmission of information, not about the conversion and encryption of data and information signals, but about the conversion and encryption of information, as well as about it automatic processing using computers. Is there a contradiction here?

Such a contradiction does exist, but it is resolved by introducing the concept of a ***context method***. Contextual is the method that is generally accepted for working with data and information signals of a certain type. This method should be known to both the creator of the message and the consumer of the information.

For illustrations (graphic, visual data), the observation method based on vision is contextual. In such cases, we are talking about graphic, figurative or *visual information*. For textual data, a contextual reading method is implied, based on vision and on knowledge of the language and alphabet. In such cases, they speak of *textual information*.

For signals represented by radio waves, the contextual methods are hardware methods for converting signals and reproducing information using a radio receiver (*audio information*) or a television (*television (video) information*).

Computer information has its own characteristics. For data presented in digital form and stored on magnetic (and other) media or for digital signals circulating in computer networks, the *hardware and software methods of computer technology* that allow processing huge amounts of information in a short period of time and also establish quantitative estimates are contextual. characteristics of the amount of information. They are also called ***means of digital information technology.***

1.4. Units of measurement of the volume of computer (digital) information.

An information signal as a storage medium can be represented in analog and digital (discrete) form. An analog signal is a process that continuously changes over time, and the value of its parameters can smoothly change, taking any intermediate values in the range from minimum to maximum. Unlike analog, a digital signal carries information about the value of the initial parameter only at fixed times, and its values themselves can also take only strictly fixed discrete values, which are usually presented in the form of binary numbers. Representation of analog signals in digital form (analog-to-digital conversion), when fixed values of the original signal, taken at certain intervals, are presented in the form of binary numbers is very convenient because it allows you to apply mathematical processing to such a signal using modern computer technology,

Analog-to-digital conversion is a complex two-part process. The first procedure is fixing the process at regular intervals, called *discretization*. The second procedure is to bring the value of a fixed parameter to one of the allowed discrete levels, called *quantization*. In practice, there are various algorithms for these procedures carried out in one sequence or another. After conversion, discrete values are replaced with a specific binary number (depending on the specific value of the discrete level). This process is called *coding*.

Information coding is not only the domain of computer technology. We very often encounter this phenomenon.

Not so long ago we used the telegraph (this service remains to this day). In this case, the sent text is encoded in the form of sequences of so-called “dots” (short signals) and “dashes” (long signals), sent by wire. At the output, all this is decoded and printed on tape.

Many people in the recent past were required to know this encoding, otherwise called the "Morse Code" by the name of its inventor.

In music, information has been encoded for many centuries using musical notation (notes). Mathematical formulas are used in mathematics. In chemistry, chemical formulas are used. There are a lot of such examples of coding information.

Compared to the above examples, the encoding used for computers looks much simpler, since it uses only “zeros” and “ones”.

The comparative simplicity of coding provides all the variety of information presented in a computer (from simple texts to complex graphic games and video films). This is due to the highest speed computers and their ability to almost instantly process huge amounts of digital data.

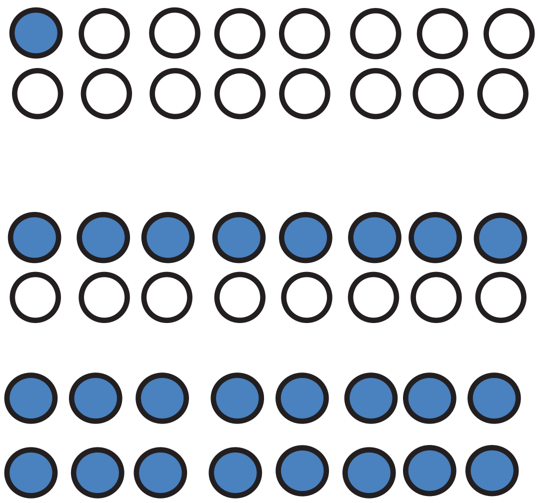
The issue of representing and encoding information in a computer is also a very important issue for establishing units of measure for the amount of information. Computers usually work in a ***binary number system***, consisting of two digits 0 and 1. The usual decimal number system consists of ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. By the way, numbers 10 is not in this list: it consists of 0 and 1 - numbers included in the decimal number system. All necessary conversions (to the decimal form familiar to us or, conversely, to the binary number system) can be performed by programs running on a computer. This means that all computer elements clearly recognize only two states: on or off, whether there is a signal or not a signal. In order to encode these two states, two digits are enough: 0 (no signal) and 1 (there is a signal). Thus, using a combination of 0 and 1, the computer (from the first generation to the present day) is able to perceive any information: texts, formulas, sounds and graphics.

One binary sign – 0 or 1 – is called a ***bit***. A bit represents the smallest unit of information. However, the computer is not dealing with individual bits, but with bytes.

Byte – a number of eight bits (various combinations of eight zeros and ones), for example,

0	**1**	**1**	**0**	**0**	**1**	**0**	**1**

A byte is a unit of measurement for digital information. One byte is capable of registering $256 = 2^8$ different combinations of eight zeros and ones.



Bit is a smallest unit of information: zero or one (“0” or “1”)

Byte consist of eight bits

Two bytes or sixteen bits

A sequence of bytes can encode text, image, sound or any other information. This method of presenting information is called ***binary coding.***

To digitize *musical sound*, you can use a device such as an analog-to-digital converter. It from the input sound (analog) signal at the output gives a sequence of bytes (digital signal). The reverse translation can be done using another device - a digital-to-analog converter, and thus play back the recorded music. In fact, the role of converters (analog-to-digital and digital-to-analog) is performed by special computer programs. Therefore, when using a computer, there is no need for such devices.

Textual information is similarly processed. When you enter into the computer each letter and each character (numbers, punctuation, space, mathematical characters, etc.) encoded so that one character occupied 1 byte of memory (eight bits, a combination of 8 ones and zeros). One letter – one character. One digit is also one character. One punctuation mark (either a period, a comma, or a question mark, etc.) is again one character. One space is also one character. So, for example, for a book of 100 pages containing 50 lines and 60 characters on each page, for its encoding, an information volume of 50 • 60 • 100 = 300,000 bytes will be required. And when displayed on the monitor screen or on the printer by these bytes, the corresponding images of text characters are readily reproduced, understandable to man.

You can save not only text and sound information. *Images* are also stored as codes. If you look at the picture with a magnifying glass, you can see that it consists of dots (pixels) of the same size and different color - this is the so-called ***raster***. The raster computer image on the monitor consists of pixels arranged in rows and columns. A pixel is an indivisible object of a rectangular or round shape, characterized by a certain color, brightness and possibly transparency. The more pixels per unit area an image contains, the more detailed it is. The pixels displayed on color monitors consist of triads (sub pixels of red, green and blue, located next to each other in a certain sequence).The coordinate of each pixel can be remembered as a number, the color of the pixel is another number for subsequent encoding. These numbers can be stored in computer memory and transmitted at any distance. According to them, computer programs are able to reproduce a picture on a monitor screen or print it on a printer. The image can be enlarged or reduced, made darker or lighter. It can be rotated, tilted, stretched. If, for example, a raster black and white picture consists of 1000 by 1000 pixels, and each pixel can be encoded with 24 bits or 3 bytes (2 bytes for the coordinates of the pixel and the third byte for its transparency), then the picture takes an information volume equal to 1000 • 1000 • 3 = 3,000,000 bytes. If the picture is colored, then the number of bytes for encoding the pixel, taking into account sub pixels increased.

We believe that the computer is processing the image. But in fact, computer programs change the numbers by which individual image points are represented (more precisely, stored) in the computer's memory.

Thus, a computer can only process information presented in numerical form. All other information (sounds, images, instrument readings, etc.) for processing on a computer must first be converted to numerical form using appropriate computer programs.

Measurement of computer information involves the consideration of other, larger units of information:

1 KB (1 ***Kilobyte***) = 2^{10} bytes = 1024 bytes (approximately 1 thousand bytes - 103 bytes)

1 MB (1 ***Megabyte***) = 2^{20} bytes = 1024 kilobytes (approximately 1 million bytes - 106 bytes)

1 GB (1 ***Gigabyte)*** = 2^{30} bytes = 1024 megabytes (approximately 1 billion bytes - 109 bytes)

1 TB (1 ***Terabyte***) = 2^{40} bytes = 1024 gigabytes (≈1012 bytes). A terabyte is sometimes called a *ton*.

1 Pb (1 ***Petabyte***) = 2^{50} bytes = 1024 terabytes (≈1015 bytes).

1 ***Exabyte*** = 2^{60} bytes = 1024 petabytes (≈1018 bytes).

1 ***Zettabyte*** = 2^{70} bytes = 1024 exabytes (≈1021 bytes).

1 ***Yottabyte*** = 2^{80} bytes = 1024 zettabytes (≈1024 bytes).

In the above table, the powers of two (2^{10}, 2^{20}, 2^{30}, etc.) are the exact values of kilobytes, megabytes, gigabytes. But the degrees of the number 10 (more precisely, 10^3, 10^6, 10^9 etc.) will already be approximate values, rounded downward. Thus, 2^{10} = 1024 bytes represents the exact value of kilobytes, and 103 = 1000 bytes is the approximate value of kilobytes.

Such an approximation (or rounding) is quite acceptable and is generally accepted.

The amount of information of modern digital media (hard or solid state drives) reaches a volume of several terabytes. For example, one and a half hours of video recording in good quality will require an information volume of about 4 gigabytes, and one gigabyte of text can be read by a person in his entire life.

1.5. Information process. Features of the information process in computer technology.

From the definition of information follows an important property of its dynamism. The fact is that information exists for a very short time — exactly as long as the interaction of data and methods continues during its creation and consumption. Then, the information extracted from the message can be processed, and the processing results generate new information, which is recorded in the form of new data or signals. Such a cycle of information formation from the data of its transformation and its immediate storage in the form of new data is called the ***information process***. If information exists for an extremely short time, then the information process itself lasts as long as there are storage media that represent information, and provides interaction between the sender and the consumer of information.

In other words, ***information processes*** are a targeted set of sequential operations performed by personnel (registration, transfer, accumulation, storage, processing, delivery of information), information exchange activities and communications carried out in a communication system.

In computer technology, as elsewhere, the information process proceeds during the interaction of data and methods. However, it has a peculiarity associated with the fact that some stages occur automatically, without human intervention. During these stages, the data represented by the registered information signals interacts with both hardware methods (computers and other devices) and software methods (computer programs).

Moreover, an important feature of computer programs is their dual nature. On the one hand, they manifest themselves as methods, and on the other hand, as data.

Computer programs can exist in two phases: active and passive. In the *active phase*, the program works in conjunction with equipment, its teams control the computer processor, which, under their influence, processes data and interacts with other equipment.

In passive phase a computer program is no different from data. It can be stored in the same way, transported through communication channels, and reproduced in the form of printed text or screen images. It can even be processed by other programs. A program presented as data can be *edited,* that is, changed its contents.

1.6. Information Properties.

So, information is a dynamic object that is formed at the time of the interaction of objective data and subjective methods. Like any object, it has properties. ***Property*** *– are the external manifestations of the process by which knowledge about the object is obtained, observation is conducted over it.* The properties of the object are perceived with the help of the senses and technical means. Properties provide the ability to describe system objects quantitatively, expressing them in units, having a certain dimension. Properties of objects may change as a result of its action.. A characteristic feature of information that distinguishes it from other objects of nature and society is the dualism noted above: the properties of information are affected by both the properties of the data that make up its content and the properties of the methods that interact with data during the information process. At the end of the process, the properties of the information are transferred to the properties of the new data, that is, the *properties of the methods can go over to the properties of the data.* This is a very important circumstance. It must be remembered that objective and reliable data at first glance may not turn out to be due to the fact that at certain stages of the information process biased or inadequate methods were applied to them.

There are many properties of information that are subjectively implied by a person as characteristics of its quality. Some of these properties of information are as follows:

Adequacy of information. Adequacy is understood as the degree of correspondence of the information received by the consumer to what the author has invested in its content (that is, in the data). Since information is a product of the interaction of data and methods, its properties, including its adequacy, are affected by both the adequacy of the data and the adequacy of the methods. The adequacy of information is sometimes mistakenly confused with its accuracy. These are completely different properties. We can give an example of adequate, but false information. So, for example, if on April 1 a deliberately false message appears in the newspaper, then it can be considered adequate. Adequately interpret it not as informational, but as entertaining. The same message, published on April 2, will be both inaccurate and inadequate. Examples of a different approach to the adequacy and reliability of information we can find in the legislation. The law distinguishes between the rights of witnesses and suspects. While the reporting of knowingly false data by the suspect is considered adequate behavior, the same actions by the witnesses are not adequate and are considered an offense.

The accuracy of the information. The reliability of information is understood as its correspondence to objective reality (both current and past) of the surrounding world. The reliability of the information is affected by both the reliability of the data and the adequacy of the methods used to obtain it. As a rule, the more initial data we have, the higher the reliability of the result. Thus, the reliability of information is influenced by its properties such as adequacy and completeness. The property of reliability of information is of particular importance in those cases when it is used for decision-making. Inaccurate information can lead to decisions that have negative economic, social and political consequences.

The completeness of the information. The completeness of information is understood as its sufficiency for making a decision. It depends both on the completeness of the data and on the availability of the necessary methods. The concept of data completeness is faced by everyone who has to perform official tasks. If the source data is incomplete, making the right decision is not easy. However, there are times when the data is complete, but we still cannot get the right solution. This indicates that we do not have any methods. And in both cases, we can say that there is not enough necessary information.

Redundancy of information. This property the usefulness of which we feel very often. Often, a person purely psychologically perceives information redundancy as its quality, because it allows him to strain his attention less and get tired less. Plain text printed in Russian has a redundancy of about 20-25%. Try to drop every fifth letter and you will see that you get info printed text is still possible, although reading it will be very tiring. We often have to deal with sloppy handwriting. The redundancy of information contained in the text provides good service, allowing you to guess the meaning of illegible characters. The visual information that we receive by the organs of vision has a very large redundancy - more than 90%. This means that, even having lost a significant part of the visual information, we can still understand its content, although not without concentration. People who are deprived of a large share of their vision continue to be full members of society, but experience increased fatigue. Even more redundancy is video information (up to 98-99%). This redundancy allows us to divert attention, which is often perceived as rest when watching a movie. Other properties are associated with redundancy of information. The higher the redundancy of the data, the wider the range of methods by which adequate information can be obtained from them. The redundancy of information allows to increase its reliability through the use of special methods, including those based on probability theory and mathematical statistics. The general principle here is this: as a result of screening, the amount of data is reduced, but their reliability increases. Of particular importance is the redundancy of information in information technologies focused on automatic data processing. On the one hand, this property is considered negative, because if the information occupies a larger volume than it could, then this leads to direct costs for its storage and, most importantly, for transportation. In such cases, it is reduced. Any reduction in data redundancy necessarily entails a reduction in the range of possible methods for their use. In information technology, the issue of data redundancy and method sufficiency is always a matter of delicate and difficult balance.

Objectivity of information. The concept of objectivity of information is relative. This is understandable given that the methods are subjective. More objective is considered to be the information into which the methods introduce a smaller subjective element. So, for example, it is generally accepted that as a result of observing a photograph of an object, more objective information is generated than as a result of observing a picture of the same object made by a person. During the information process, the degree of objectivity of information always decreases. This property is taken into account, for example, in legal processes, where the testimonies of persons directly observing events and persons who received information indirectly are processed differently (through inferences or from the words of third parties). To a no lesser degree the objectivity of information is taken into account in historical disciplines. The same events recorded in historical documents of different countries and peoples can look completely different. Specialists have their own methods for testing data objectivity and creating new, more reliable data by comparing, filtering and selecting source data. We draw attention to the fact that here we are not talking about increasing the objectivity of information, but about increasing its reliability (this is a completely different property).

Information Availability. Accessibility of information is a measure of the ability to receive this or that information. The degree of accessibility of information is affected simultaneously by both the availability of data and the availability of adequate methods for their interpretation. Lack of access to data or lack of adequate data processing methods lead to the same result: information is not available. The lack of adequate methods for working with data in many cases leads to the use of inadequate methods, resulting in the formation of incomplete, inadequate or inaccurate information.

Relevance of information. Relevance is the degree to which information corresponds to the current point in time. Often relevance, as well as completeness, is associated with the commercial value of information. Since information processes are stretched in time, reliable and adequate, but outdated information can lead to erroneous decisions. The need to search (or develop) an adequate method for working with data can lead to such a delay in obtaining information that it becomes irrelevant and unnecessary. In particular, many modern data encryption systems and electronic signature mechanisms are based on this. Persons who do not own a key (method) for reading data can search for a key, since the algorithm of the method is usually available, but the duration of this search is so long that during operation the information loses its relevance and, accordingly,

1.7. Data, their media and processing.

Data – dialectical component of information. They are registered information signals. In this case, the physical registration method can be any: mechanical movement of physical bodies, change in their shape or surface quality parameters, change in electrical, magnetic, optical characteristics, chemical composition and (or) the nature of chemical bonds, change in the state of the electronic system, and much more.

In accordance with the registration method, data can be stored and transported on various types of media. The most common storage medium, although not the most economical, is apparently a paper. Data is recorded on paper by changing the optical characteristics of its surface. A change in optical properties (a change in the surface reflection coefficient in a certain wavelength range) is also used in devices that record with a laser beam on plastic media with a reflective coating (CD-ROM). As carriers using a change in magnetic properties, magnetic tapes and disks can be mentioned. Data recording by changing the chemical composition of the surface substances of a carrier is widely used in photography. At the biochemical level, the accumulation and transmission of data in wildlife occurs.

Storage media are not of interest to us on their own, but insofar as the properties of information are very closely related to the properties of its carriers. Any medium can be characterized by a *resolution parameter* (the amount of data recorded in the unit of measurement adopted for the medium) and a *dynamic range* (logarithmic ratio of the amplitudes of the amplitudes of the maximum and minimum recorded signals). Information properties such as completeness, availability and reliability often depend on these media properties. So, for example, we can count on the fact that it is easier to ensure the completeness of information in a database placed on a CD than in a similar purpose database located on a floppy disk, since in the first case the data recording density per unit length the paths are much higher.

The task of converting data in order to change the media is one of the most important tasks of computer science. In the structure of the cost of computing systems, devices for input and output of data working with storage media make up half the cost of hardware.

During the information process, data is converted from one form to another used methods. Data processing involves many different operations. With the development of scientific and technological progress and the general complication of relations in human society, the labor costs of processing data are steadily increasing. First of all, this is due to the constant complication of the conditions for managing production and society. The second factor, which also causes a general increase in the volume of processed data, is also associated with scientific and technological progress, namely, with the fast pace of the emergence and implementation of new data carriers, data storage and delivery tools.

In the structure of possible data operations, the following main ones can be distinguished:

- *data collection* – accumulation of data in order to ensure sufficient completeness of information for decision-making;
- *data formalization* – bringing data from different sources to the same form in order to make them comparable, that is, to increase their level of accessibility;
- *data filtering* – sifting out “extra” data that is not necessary for decision-making; at the same time, the level of “noise” should decrease, and the reliability and adequacy of the data should increase;
- *data sorting* – ordering data according to a given attribute for the purpose of ease of use; increases the availability of information;
- *data grouping* – combining data on a given basis in order to improve usability; increases the availability of information;

- *data archiving* – organization of data storage in a convenient and easily accessible form; serves to reduce the economic costs of data storage and increases the overall reliability of the information process as a whole;
- *data protection* – a set of measures aimed at preventing the loss, reproduction and modification of data;
- *data transportation* – reception and transmission (delivery and delivery) of data between remote participants in the information process; while the data source in computer science is usually called the server, and the consumer is called the client;
- *data conversion* – transferring data from one form to another or from one structure to another. Data conversion is often associated with a change in the type of medium, for example, books can be stored in ordinary paper form, but you can use both electronic form and microfilm for this. The need for multiple data conversion also arises during their transportation, especially if it is carried out by means not intended for the transportation of this type of data. As an example, we can mention that for the transportation of digital data streams over telephone network channels (which were originally oriented only to the transmission of analog signals in a narrow frequency range), it is necessary to convert digital data into a kind of sound signals, which is what special devices – *telephone modems* do.

The list of typical data operations given here is far from complete. Millions of people all over the world are engaged in the creation, processing, transformation and transportation of data, and at each workplace they carry out their specific operations necessary to manage social, economic, industrial, scientific and cultural processes. A complete list of possible operations is impossible, and not necessary. Now another conclusion is important to us: *working with information can be enormous laborious, and it needs to be automated*. For this purpose, information systems are currently designed.

TOPIC 2. ORGANIZATIONAL AND METHODOLOGICAL FOUNDATIONS OF THE CREATION AND FUNCTIONING OF INFORMATION SYSTEMS

2.1. A systematic approach to the analysis of information processes.

So, as already said ***information processes*** – it is a targeted set of sequential operations performed by personnel on registration, transfer, accumulation, storage, processing, delivery of information, actions and communications for the exchange of information carried out in a communication system, starting from the moment of its receipt from the source of information (occurrence), and ending with the issuance to the user. Moreover, the concept of "user" must be understood in a broad sense. The user can be: a person receiving information in a convenient form for perception; technical devices, information converters on magnetic tape, disk, in random access memory, communication channel, etc .; programs that convert the information received.

To analyze information processes, we will use a systematic approach.

Systems approach *– this is the methodology of scientific knowledge and practical activity, as well as the explanatory principle, which is based on the consideration of the object as a system.*

A systematic approach is the rejection of one-sided analytical, linear-causal research methods. The main emphasis in its application is on the analysis of the integral properties of the object, the identification of its various relationships and structures. A systematic approach seems to be the most universal method of analysis and research of any complex technical, economic, social, environmental, political, biological and other systems.

A system can be any object of animate and inanimate nature, society, a process or set of processes, scientific theory, etc., if they define ***elements that form a unity (integrity) with its connections and interconnections between them***, which ultimately creates a set of properties inherent only to this system and distinguishing it from other systems. In other words, it means that the properties of the system, although they depend on the properties of the elements, are not completely determined by them. ***Conclusion: the system is not reduced to a simple set of elements, and, dividing the system into separate parts, it is impossible to know all the properties of the system as a whole.*** This property of the system is called ***emergence,*** a synonym is the ***system effect.***

Element – indivisible part of the system, which has independence in relation to this system. The indivisibility of an element is considered as the inadvisability of accounting within the model of a given system of its internal structure. The element itself is characterized only by its external manifestations in the form of connections and interconnections with other elements.

Under certain conditions, elements of a system can in turn be considered as systems, in which case they are called subsystems with respect to the system containing them.

Subsystem – *part of the system, which in turn acts as a system of a lower order, the purpose of which is determined by the functions of a system of a higher order and hierarchy*.

Hierarchy – a *structure characterized by the presence of subordination*, i.e., unequal relationships between elements, when the impact in one direction has a greater effect on the element than in the other.

The system exists among other material objects (systems) that are not included in it, but interact with it. They are united by the concept of "***external environment***". In fact, the delineation or identification of a system is the division of a certain area of the material world into two parts, one of which is considered as a system – an object of analysis (synthesis), and the other – as an external environment. ***The relationship of the external environment and the system*** can be considered one of the main features of the functioning of the system, an external characteristic of the system, which largely determines its properties.

Thus, in the most general case, taking into account the foregoing the term "*system*" is used in two ways:

• as some method, the essence of which is the rational unification and ordering of all elements in time and space by the researcher in such a way that each of them contributes to the success of the activity of the whole object to achieve a certain goal. In this sense, for the term "system" we can give the following definition:

***System** – a complex of interconnected parts, elements or subsystems united by a researcher to achieve a certain goal or set of goals. An additional feature of the system is the belonging of each element and each connection between the elements to the fulfillment of its objective function;*

• as an object that has a rather complex, ordered internal structure. With this approach, for the term "system, we come to the following definition:

***System** – this is the set of elements that make up the unity of elements, their connections and interactions between themselves and between them and the external environment, forming the integrity inherent in this system, qualitative certainty and purposefulness.*

Based on the above concepts and definitions, we now define the information process as an information system.

2.2. Information systems, their structure and purpose.

According to a systematic approach, the information process can be represented as an organized set of elements united by regulated relationships performed by personnel for purposeful activities to provide the consumer with the information he needs (see, for example, Fig. 1) and, therefore, can be considered as a system – an information system. Based on this, we come to the following definition:

***Information system** – interconnected set of tools, methods and personnel used to collect, store, process, transform, transmit and issue information in the interests of achieving the goal.*

The structure of any information system can be represented by a set of supporting subsystems. Among the supporting subsystems, information, technical, mathematical, software, organizational and legal support are usually distinguished.

Information support is the totality of a unified system of classification and coding of information, unified documentation systems, information flow schemes circulating in the system, as well as a methodology for building databases.

Technical support – a set of technical tools intended for the operation of the information system for the collection, storage, processing, transmission and delivery of information, as well as the relevant documentation for these tools and technological processes.

Mathematical and software – a set of mathematical methods, models, algorithms and programs for implementing the goals and objectives of the information system, as well as the normal functioning of a complex of technical means.

The means of mathematical support include:

- tools for modeling management processes;
- typical management tasks;
- methods of mathematical programming, mathematical statistics, queuing theory, etc.

Organizational support – a set of methods and tools governing the interaction of workers with technical means and among themselves in the process of developing and operating an information system.

Legal support – a set of legal norms that determine the creation, legal status and functioning of information systems governing the procedure for obtaining, converting and using information.

To ensure the operation of the information system for any purpose, you must

- identify information needs;
- select sources of information;
- implement the introduction of information into the system, both from external and internal sources;
- perform actions to process information, assess its completeness and significance and to present it in a convenient form;

- display information for provision to consumers or transfer to another system;

- organize the transformation of information in order to use the information to assess trends, develop forecasts, to develop and evaluate alternatives to managerial decisions and actions, develop strategies;

- organize ***feedback*** – according to the output information, correct the input information in order to maintain the system in a given state (***negative feedback***), or to put the system in the desired state (***positive feedback***), or organize both positive and negative feedback.

The basic components of an information system for any purpose are:

- information resource (database);

- information Technology.

Information resource (***database***) – the central element of the information system, which is data containing information about the activities of the object.

Any information system is characterized by the presence of technology for converting source data into effective information. Such technologies are called information.

Information technology – it is a complex of methods and procedures by which the functions of collecting, transmitting, processing, converting, storing and communicating information to a user in organizational and management systems using the selected set of technical means are implemented. In other words, ***information technology*** *is a system-organized sequence of operations performed on data to receive, transform and transport information using automation tools and techniques.*

In information technology, two parts can be distinguished:

1. Ability to generate information product upon request;
2. The means of delivery of this information product at a convenient time and in a user-friendly form.

The implementation of information systems (IS) changes the management technology, frees users from routine, rather simple, but labor-intensive manual procedures for processing information, and the effectiveness of such implementation depends on the applied information technologies. IP use information technology software and hardware, the content of which is constantly expanding and improving, which leads to the improvement of IP. Currently, the development of IP is due to the use of digital information technologies in computer systems (see section 1.3), including multimedia and telecommunications. Computer information systems using digital information technology provide the accumulation and processing of information for the purpose of analysis and the creation of a decision support system of rather large volumes, which a person is not able to perform.

Currently, depending on the use of technical means, there are two ways of processing information - the combination of methods and technical means by which information is processed: without using computers and using computers.

Processing information **without using computers has advantages**:

- minor requirements for the qualifications of employees;

- the complexity of accidental loss of information and good security of information from unauthorized interference;

- high legal evidence of primary documents drawn up.

Disadvantages:

- significant duration and complexity of the process of collecting and processing information;

- low level of detail information;

- significant volumes of routine operations.

Information processing **using computers has advantages**:

-the possibility of a wide division of labor between workers;

- the ability to multiple users to work with the same register;

- the possibility of significant data granularity in the analysis of information;

- automatic control over arithmetic calculations, combining documentation and recording in accounting registers;

- significant speed of obtaining effective information, the ability to maintain a multi-dimensional and multi-level data array;

- reducing the flow of paper documents and the transition to paperless technology, the ability to daily generate enterprise reporting as necessary;

- the ability to choose the form of presentation of information, the ability to search for information at their workplaces in order to save working time.

Disadvantages:

-insufficient solution to the problem of protecting information from unauthorized interference;

- the problem of legal evidence of compiled electronic primary documents;

- the possibility of loss or damage to information during a power outage, the penetration of computer viruses;

- significant one-time costs for the implementation of automation, the acquisition of computer equipment, software and staff training.

2.3. Computer information systems and their classification.

Modern ***computer information system*** *– a set of economic and mathematical methods and models, technical, software, technological tools and solutions, as well as specialists, designed to process information and make management decisions using computer and computer technology* has wide capabilities that specialists are not yet fully took advantage of. Intensive work is underway to develop these opportunities, primarily with the involvement of the Internet and putting them into practice. Computer information systems play a significant role in modern enterprises. They have a direct impact on the planning and management decisions, nomenclature and technology for the manufacture and sale of goods and services. The introduction of computer ICs is changing management technology, frees users from routine, rather simple, but time-consuming manual procedures for processing information. Computer ISs change the organizational structure, the composition of management functions and the associated information flows, the presentation form and the qualitative characteristics of information (efficiency, reliability, accuracy, completeness of information for management decisions).

Software for ***computer information technologies***, i.e. *information technologies implemented using computer technology* provide the ability to programmatically process information in electronic form, they include:

1. System software that provides the work of electronic computers and computer networks;
2. Application software that allows you to solve specific management tasks and process information in accordance with the tasks assigned to the employee and his immediate job duties.

Technical means for ***computer information technologies*** are divided into classes:

1. Means of collecting and recording information (PCs, scanners, automatic sensors);
2. Information transfer tools (local, regional and global computer and telecommunication networks, Internet);
3. Data storage facilities (database servers, file servers, optical (laser) disks, digital video disks);
4. Data processing tools (local, regional and global computer networks);
5. Means of information output (video monitors, printers, plotters).

At the same time, the large-scale goal is to create and deploy a single corporate information system meeting the information needs of all employees, services and units of the organization.

Corporate information system *– is a scalable system designed for the comprehensive automation of all types of economic activity of large and medium enterprises, including corporations consisting of a group of companies requiring a single management.*

The corporate information system integrates the personnel management system, material, financial and other resources of the company, it is used to support the planning and management of the company, to support management decision-making by its leaders. The effectiveness of a corporate information system can only be assessed by the results of its contribution to the achievement by the organization of its strategic goals.

Computer IS can be classified and vary significantly in terms of types of control objects, scope, nature and volume of tasks to be solved, and other features. An example is the following classification.

State corporate IP designed to solve the most important national economic problems of the country. Based on computer systems and economic and mathematical methods, they draw up long-term and current development plans of the country, develop the state budget and monitor its implementation, etc.

In this case, the ***problem*** refers to the mismatch between the goal achieved by the economic object and the desired one, which the management of the object would like to achieve. The ***solution to the problem*** is a set and sequence of actions that must be performed on an economic object in order to bring it into a state ensuring the achievement of the desired goal.

Regional computer information systems designed to perform the processing of information that is necessary for the implementation of the functions of managing the region, generating reports and issuing operational data to local and governing state and economic bodies.

Automated enterprise management systems (AMS) are systems using modern automated data processing, economic, mathematical and other methods to solve the problems of managing production and business activities of enterprises.

Automated process control systems (ACS TP) manage the state of technological processes.

Information retrieval systems (IRS) are focused on solving problems of finding information without its meaningful processing.

Information and reference systems (IRS) are designed to calculate the values of arithmetic functions according to the search results.

Management information systems (MIS) are designed to automatically solve a wide range of management tasks using a database management system (DBMS).

Decision Support Systems (DSS) are interactive computer systems that are designed to support various types of activities and decision making in relation to weakly structured or unstructured problems.

Expert systems are information systems that are based on knowledge and make it possible to efficiently computerize areas that are presented in expert descriptive form, and the use of mathematical models is impossible or difficult.

Thus, an information system can be technically defined as a set of interdependent components that collect, process, accumulate and distribute information to support decision-making and management in an organization. In addition to supporting decision-making, coordination and management, information systems can also help managers analyze problems and develop solutions to address them. In the automated implementation of information technology, computer programs exist.

Computer program *– a set of instructions in the form of words, numbers, codes, circuits, symbols, or in another form, expressed in the form that the computer reads, and which power it to achieve a specific goal or result.*

Programs designed for computers are divided into: ***operating systems*** (Windows XP, Windows Vista, UNIX, Linux), ***system software*** (drivers, utilities), ***application programs*** (text editors, spreadsheets, programs designed to solve specific problems), ***programming languages*** (Pascal, Delphi), other programs.

The main means of implementing computer information technologies at the workplace of an employee is his ***automated workplace (AWP)*** **–** a professionally oriented software and hardware complex that provides a solution to user problems directly at his workplace.

The requirements for software products are formed by three different categories of users: direct users of a computer system (operators), information system specialists (programmers and technicians), managers (enterprise management, managers).

The list of program requirements is a complex set of functional, technical, commercial and ergonomic issues.

The main **functional requirements** include: the ability to enter and accumulate all operations that reflect the activity of the investigated object; the ability to calculate results, draw up reporting forms based on the results of the analysis at any time with any number of operations entered; processing of correspondence and delivery of the results of the analysis of operations indicating specific analytical objects for its presentation.

Technical requirements consist, firstly, as programming (data processing algorithms), and secondly, in providing a number of service parameters that are necessary for the effective operation of the program within the hardware-software complex. The main technical requirements include: unpretentiousness to hardware – the program should work effectively on computers with average characteristics; the ability to edit documents and reports, which allows the user to create a report form, which will then be displayed on a monitor or printer; data exchange with other programs and devices; work in local computer networks; information security using a password system; means of creating data archives and automatic recovery of information in the event of a failure.

Commercial requirements include requirements that are related to the conditions for the acquisition and operation of the program that are offered by the supplier. The main commercial requirements include: an acceptable price for the program (it is necessary to distinguish between the cost of the actual software product and the cost of its implementation and maintenance): software maintenance, which involves training in working with the program and debugging the program to solve a specific problem using real user data; operational advice from problems that arise in the process of work; updating old versions of software products, quickly replacing them with new ones; high quality paper and electronic documentation.

Ergonomic requirements – convenient user interface and advanced software assistance system.

The recommended program selection sequence is as follows:

1) determination of the main activities;

2) determination of the budget for the implementation of a computer program;

3) determination of the list of programs that have acceptable functionality and meet the existing technical support and cost budget;

4) receiving demo versions of programs for familiarization with them, choosing the optimal version of the program and evaluating its suitability.

2.4. The principles of design and creation of computer information systems.

Creation of computer information systems (hereinafter simply IS) is a focused activity aimed at providing all users of information, united in a certain structure (for example, an enterprise) with the information they need.

When creating IS, different approaches are used:

1. Depending on the changes made in the organizational structure of information users, there are approaches that preserve the existing organizational structure, and approaches that change the existing organizational structure, improving it.

2. Depending on the nature of the changes made to the IS, there are improvement approaches, according to which individual processes of operations and integrated approaches that improve the IS as a whole are improved. The main means of comprehensive improvement of IS is a structural approach, which provides for the consistent distribution of the IS system into separate modules. Each of the modules will have a certain function, depending on the purpose, followed by combining the resulting modules into a single whole, which will represent an advanced IS.

3. Depending on the method of implementing the improvement of IS, such approaches to improving the IP are distinguished: improvement on their own, improvement using a standardized solution, which has been tested and ordering an individual IS – system integrator.

When designing IS, certain principles must be taken into account, the observance of which will significantly increase the efficiency of IP. The following **design principles of IS** are distinguished:

1) ***Systematic.*** Analysis of the control object as a whole and its control system, determination of common goals and control criteria, as well as determination of common goals and criteria for the operation of the object in the context of its automation. This principle provides for a one-time introduction of information into the system and its multiple use, the presence of a single information base, and integrated software. This principle is fundamental in the design of IP.

2) ***Economic feasibility***. The benefits that are expected from the use of the system should exceed the costs of designing, implementing, studying support.

3) ***Flexibility.*** The system that is being created must have a sufficient margin of flexibility to provide the ability to respond to changing external factors.

4) ***Control.*** Creating parallel information flows that control each other and ensure the reliability of the data.

5) ***Data protection and security***. The system that is being designed should ensure the reliability and security of information in the system. A high-quality computer system should perform such functions with respect to data security: low access to the functions and data of the system by authorizing users with a password; data encryption; the presence of control over the entrance to the system and the logging of working hours; control over the frequency of creating backup (archival) copies of information.

6) ***Compatibility.*** The principle of compatibility means that the system should be designed taking into account the human factor and organizational characteristics of the enterprise, existing computers and programs.

7) ***Versatility***. The software system should not solve a separate problem, but should perform standard procedures and process a specific task as a special case of a more general one.

8) ***Continuity of development*** provides for continuous improvement of all types and its support (technical, software, information, etc.). This is due to the fact that with the development of enterprises new management tasks arise, old ones improve and change.

Regardless of the approach to creating IS, adherence to the principles of designing IP ensures the creation of a highly efficient, working system that can be quickly adapted to new requirements of information users.

In conclusion, we note the main features of modern information systems. Any IS can be analyzed, built and managed based on the general principles of building systems; IS is dynamic and evolving; when building IP it is necessary to use a systematic approach; IS output is information on the basis of which decisions are made; IS should be understood as a human-computer information processing system.

The fact that IS should be understood as a human-computer information processing system means that activities to provide all users with the information they need are carried out by specially trained personnel.

Job descriptions of employees when using a computer method of processing information should include:

1. Employee access rights to the computer and information resources.
2. The list of documents with which the employee works, and the list of operations that he performs.
3. The procedure for storing paper documents.
4. The procedure and method for transferring paper documents to the information base.
5. Rules for creating electronic documents and reports.
6. Frequency of reporting.
7. The procedure for generating requests to employees of different services.
8. A method of transferring information to other jobs.

9. The procedure for storing archival information and the destruction of electronic archival copies.

10. The frequency of monitoring the workplace by specialists.

11. Frequency of checking the workplace by technical services.

As follows from the above consideration, one of the main purposes of the created IP (in addition to automation)- this is the production of the necessary information to ensure effective management of the facility to achieve any goals. A detailed review of management IP is the subject of our further discussion.

TOPIC 3. MANAGEMENT INFORMATION SYSTEMS

3.1. Information analysis and synthesis of control systems.

The object of information analysis and synthesis are information processes that occur in the control system. Let us dwell on their essence and content, for which we consider:

- communication (information) scheme for transmitting information in the control system;
- classification and characterization of information processes;
- structure of the information process.

Communication (informational) information transfer scheme in the control system. This scheme of information transfer in the control system is shown in Fig. 2.

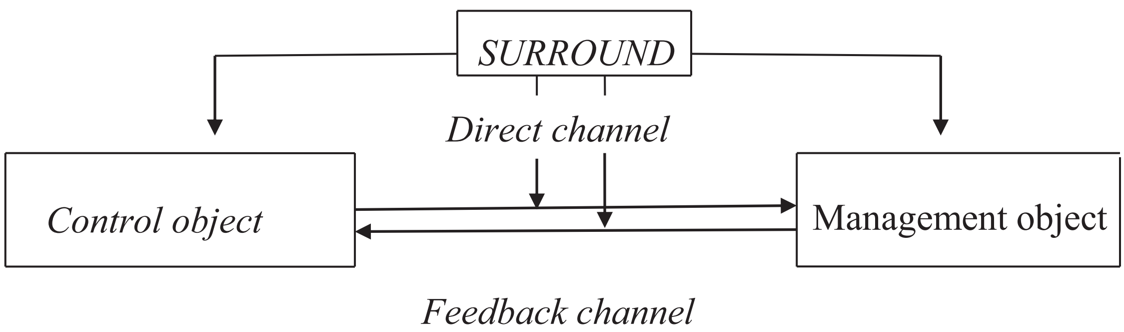


Fig. 2. Communication scheme of information transfer in the control system

In fig. 2, the source of information is a control object that sends information about its state via the feedback channel. The information receiver, it is also a control object, depending on the amount and content of information about the control object, develops a decision on the impact on it. This solution in the form of commands and control signals via a direct communication channel is transmitted to the control object. After executing the commands the control object sends information about the change in its state to the control object, etc. Thus, the information exchange cycle in the control system is closed, and for the control system the following definition can be given:

Control system – a set of stable relationships between the managing facility and the management entity, implemented in specific organizational forms to achieve the desired goals.

The task of the control body is to convert the external influence on the control object into a control action so that at the output of the control object there is some desired state.

In really functioning control systems, the information source, the information receiver, and the information transmission line are affected by the environment in which the given system functions, making its own adjustments both to the amount of information and the quality.

The management process, as the process of generating control actions, is an ***information process*** (IP).

Information process is a purposeful set of operations to transform information implemented in a particular environment, starting from the moment it enters the system (occurrence), and ending with the issuance to the user. Moreover, the concept of "user" must be understood in a broad sense. The user can be: a decision maker, receiving information in a convenient form for perception; technical devices, information converters on magnetic tape, disk, in random access memory, communication channel, etc.; programs that convert the information received.

Any information process in the control system can be structurally represented as a set of processes:

- collection, reception, perception of information (these processes reflect the interaction of the system with the external environment);
- information transfer between separate subsystems of the system;
- processing, analysis, selection of information, the creation of new information;
- use of information;
- storage, memorization of information;
- transfer of information from the system to the external environment.

Management (information) processes can be implemented on a computer if they can be formalized. Formalization refers to the description of the object being studied in exact terms or statements. Formalization is inextricably linked with the construction of artificial, or formalized, scientific languages. Such languages are created to accurately express thoughts in order to exclude the possibility of ambiguous understanding. Formalization makes it possible to build scientific languages with a precisely established structure and given rules for converting some expressions into others. An indispensable condition for constructing such a language is the use of the axiomatic method, thanks to which it is possible to obtain all the statements of the theory from a small number of statements accepted without proof, or axioms. Formalization of the management process includes the allocation of managerial functions and tasks, the development of formalization algorithms and the implementation of algorithms. The process is considered completely formalized,

For the convenience of research, it is advisable to classify them. IP classification is carried out both in the analysis and in the synthesis of control systems.

Classification and characterization of information processes. In the classification, it is necessary to distinguish between homogeneous and heterogeneous sets of PIs.

We will consider a given set of IPs to be homogeneous if the nature of the use of the resources of the system ensuring their implementation is the same for all IPs and the order of service in it is not related to their belonging to specific IPs. Otherwise, the set of PIs is considered heterogeneous.

In turn, among heterogeneous sets of IPs, priority and non-priority ones can be distinguished. A heterogeneous set of IPs is non-priority if the procedure for servicing applications of individual IPs does not depend on their belonging to specific IPs, otherwise it is considered a priority.

The following service disciplines are most characteristic of a non-priority set of individual entrepreneurs: in the order of receipt; in reverse order; based on time sharing.

For a priority set of IPs, the discipline of service depends on the importance of IPs. The service procedure (provision of resources for the implementation of IP) is based on an analysis of the priorities of the IP, for which each IP is assigned a number called the priority. It is generally believed that the smaller the number, the more “important” the IP, that is, the greater the advantage it should be given.

By the method of assigning priority, systems with fixed and dynamic priorities are distinguished. In the first case, the numerical value of the priority of the IP remains constant during its maintenance (including taking into account the waiting time for the service), in the second case, it changes over time, being a function of the service time.

A priority population that does not belong to any of these types is called composite. For her, various combinations of the described methods for assigning priority to “competing” IPs are usually used.

Despite the variety of IPs flowing in control systems, from the point of view of information processing technology, they have much in common. This allows us to present a generalized scheme of information processing in control systems (Fig. 3).

Sources of information in control systems can be *officials* (OP) of management, *automatic sensors* (AS) and *computer systems* (CS). In general, the information passes to the following main technological stages of conversion until it is issued to the user:

- data collection (manually or automatically);
- formation of a message (request);
- data transmission via communication channels using automation tools or traditional methods;
- delivery of data to the decision maker or for their input into the computer complex (CS);
- solving informational or computational problems in CS;
- delivery of the results of solving OP problems;
- Bringing the decision or tasks to the addressees.

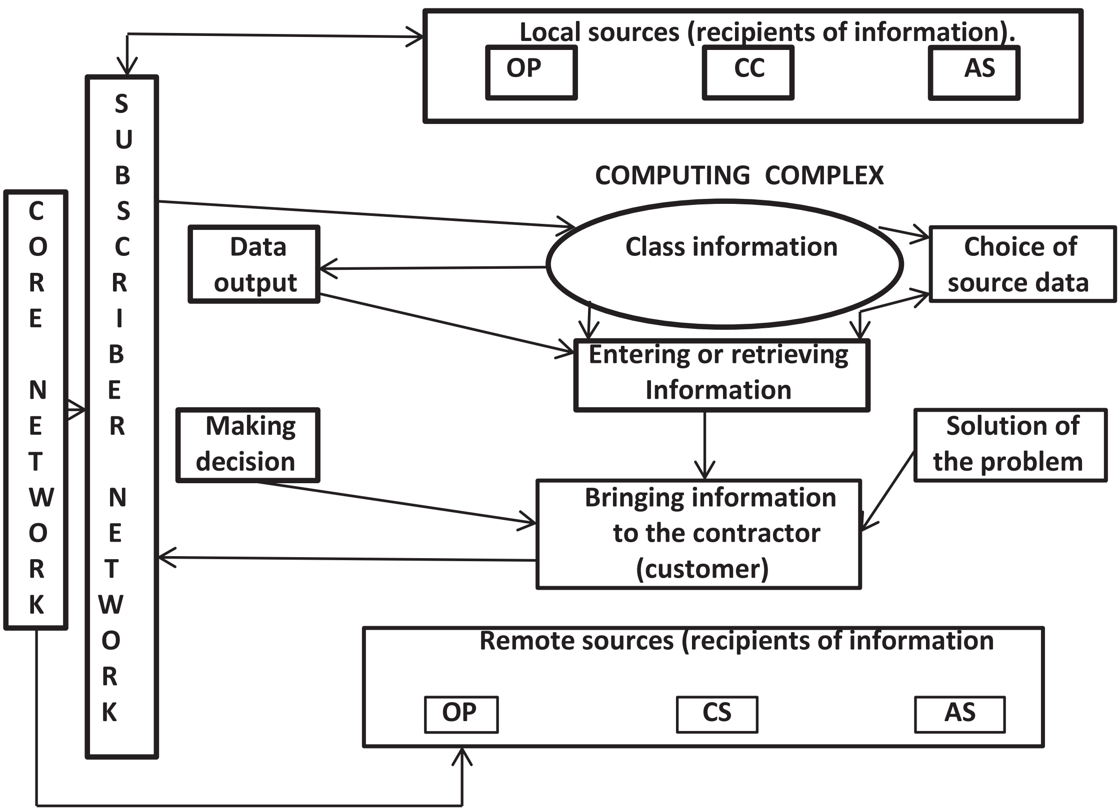


Fig. 3. Generalized information processing scheme in control systems

Sources of information in control systems can be *officials* (OP) of management, *automatic sensors* (AS) and *computer systems* (CS). In general, the information passes to the following main technological stages of conversion until it is issued to the user:

- data collection (manually or automatically);
- formation of a message (request);
- data transmission via communication channels using automation tools or traditional methods;
- delivery of data to the decision maker or for their input into the computer complex (CC);
- solving informational or computational problems in VK;
- delivery of the results of solving DM problems;
- Bringing the decision or tasks to the addressees.

For different categories of users, the number of stages, their duration, as well as the types of messages (requests) generated by them, as a rule, are different. They depend on the specific purpose of the system, the technical implementation of its individual elements, their priorities, the nature of the information transmitted in information messages, etc.

So, to get help, users locally connected to a sufficiently powerful CS can attract other CSs (that is, without going to the data transfer system).

The structure of the information process. Having defined the concept of the information process as a set of operations for the conversion of information, the following should be noted.

Any information unit (I) with consumer value (quality) is characterized by content (S), form (F), spatial location (L) and time (T), i.e. $I = \{S, F, L, T\}$.

Each of these characteristics in the process of transformation (in the information process) can change. The following types of information conversion are distinguished:

- meaningful (semantic processing, as a result of which new information is obtained);

- form conversion (e.g., encoding, decoding, etc.);

- transformation in space and time (for example, data transmission and, accordingly, storage).

The elementary action in the information process is the operation of information conversion (Fig. 4).

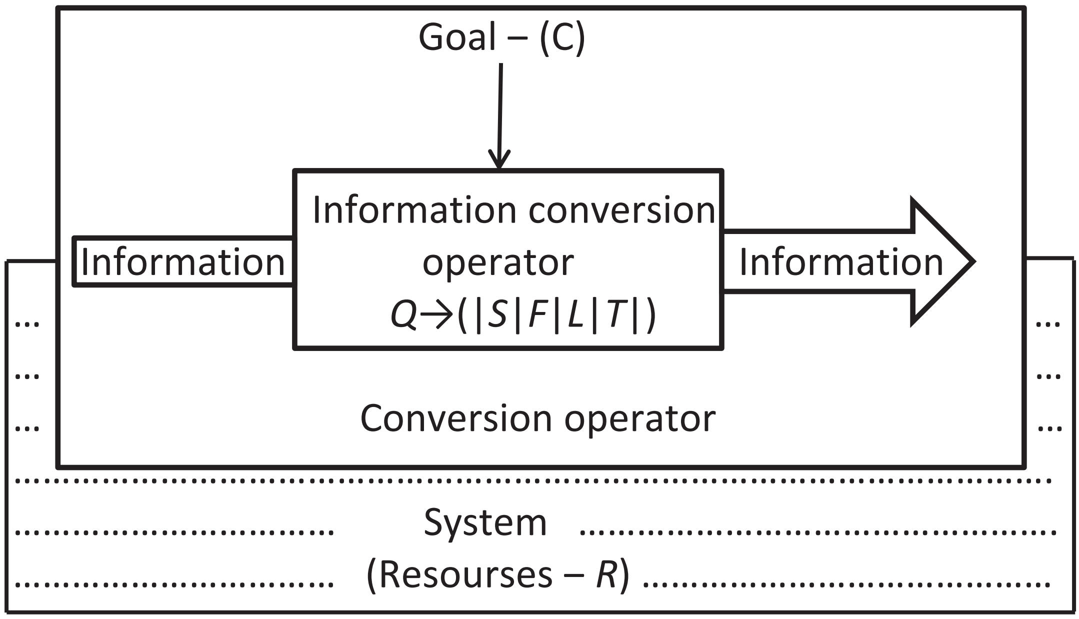


Fig. 4. The structure of the information conversion operation

The main attributes of the operation are information (transformation object); operator (subject of transformation) and purpose (requirements for transformation).

Information conversion operations can be of different types (V), which are determined by the type of operator $Q \rightarrow (| S | F | L | \mathrm{T} |)$, complexity ($U$), depending on the type of operator and the purpose of the conversion $U = <V, C>$, time (T) of its implementation $\mathrm{T} = <U, I>$ and resource consumption (R).

The information process can be structurally represented by many sub processes. Depending on what types of information transformation dominate (i.e., are the main ones in the system of achieving the goal), the following sub processes can be distinguished.

1. *Formation process* (*preparation*) *of information* for transformations. The dominant type of transformation is the transformation of the form (F).

2. *Information transfer process* from source to consumer. Obviously, the

main type of transformation here is the transformation of information in space (*L*).

3. *Semantic process* (*semantic*) *information processing* – central IP sub process in the control system. As a result of its implementation, new information appears, on the basis of which, ultimately, control actions are formed. It is through this process that the entire system with control can be fully identified, and the effectiveness of the functioning of the system depends on the quality of information processing. The purpose of this process is the development of adequate control actions in an optimal (rational) way. The dominant type of transformation in this case is the transformation of the content of information (*S*).

4. *The process of storing information.* The purpose of this process is to ensure the existence of information in time. The main type of transformation here is the transformation of information in time (*T*).

Obviously, all real processes and systems that implement them, and, consequently, IP structures, are unique. The specifics of the functioning of real systems (which determines their uniqueness) is manifested in the content of the information itself, the composition and sequence of application of specific operators for its transformation and goals, the essence of a process, the composition and relationship of sub processes, as well as in the specific global goals of the functioning of the system with control.

Management (information) processes can be implemented on a computer if they can be formalized. Formalization refers to an accurate description of the object being studied. Formalization of the management process includes the allocation of managerial functions and tasks, the development of formalization algorithms and the implementation of algorithms. The process is considered completely formalized if the algorithms for the tasks are presented and mathematically translated into machine programs, and when solving problems, their physical content is no longer necessary. The need for knowledge of physical content appears only when using the results.

Information analysis performed to study the quantitative and qualitative characteristics of the information used in the control system.

The objects of research are information processes that occur in the control system.

The information analysis procedure includes:

- determination of the need for information at each stage of management;
- information needs planning;
- determination of quantitative and qualitative characteristics of communication processes;
- determining the need for information in assessing the effectiveness of managerial decisions (impacts).

The indicators (characteristics) of information include:

- volume and speed of information transfer;
- the reliability of the transmitted messages;
- direction of information flows;
- characteristics of information processing methods and errors made in doing so;
- high-quality information;
- the number of processed or transmitted documents;
- the total number of processed or transmitted documents, etc.

Based on the results of the information analysis, preliminary recommendations are developed on the development of information support for the management system, including methods for transmitting, processing and presenting information, the composition of the information necessary for the normal functioning of the system, the structure of information exchange, etc.

In a systematic sense, information is considered in terms of the following aspects:

pragmatic – fulfillment of goals and objectives, criteria for assessing their feasibility, ways to describe information images and flows, strategies and tactics for achieving goals;

semantic – semantic content of information, its unambiguity and concreteness and applicability in fulfilling the goals and objectives;

syntactic – the rules and laws of obtaining, processing, storage and transmission of information, and includes the structural aspect - the rules and laws of classification of information, the formation of information files, databases and knowledge bases.

The essence of information synthesis is the justification of the necessary volume and forms of presentation of information, methods and means of its transmission, processing, storage, input and output for the developed structure and algorithm of functioning of the control system.

Information synthesis supplements the tasks of functional synthesis and is carried out in order to determine the required qualitative and quantitative characteristics of the information used in the process of functioning of the control system.

In the course of information synthesis, the substantiation of the necessary volume and forms of the presentation of information, methods and means of its transmission, processing, storage, input and output is made.

Optimal or rational characteristics of information are determined, as a rule, using indicators and performance criteria that take into account the structural and functional features of the system. In this case, one of the main problems of information synthesis is the quantitative assessment of the influence of these characteristics on the results of the functioning of the control system.

3.2. The concept of "black box" in the information analysis of systems.

As we already know, the object under study, considered as a system, is an integral part of a number of different hierarchical systems. In economic systems, the main actor is a person. Therefore, when studying a specific economic problem, we are forced to “break off” the hierarchy of systems “going down” at some stage. Whether we do it on a person, or on some set of people – it already depends on the problem being studied.

Of course, a person does not necessarily act as the “smallest” element that can be considered as “indivisible”. It may well turn out that as such an “indivisible” element, one can consider, for example, individual firms (for the tasks of optimizing the management of the regional economy), social groups (for the tasks of allocating state budget funds), or sectors of the economy (when considering the balance of resources within the framework of gross domestic product).

In other words: at some stage of the investigation, the elements that make up our system are not assumed by us as systems, but as “finite” and “indivisible” objects. Thus, the hierarchy of systems unfolds upward, proceeding from such objects, which thereby become objects of the lowest level of the hierarchy.

Such an object – by virtue of our assumptions (that is, from our *situational* point of view) – will no longer have an “internal structure”. Therefore, it should be considered as an object that can be characterized - within the framework of the problem we are considering – by only two classes of characteristics. The need for this arises due to the reason that such objects must form a system – that is, they must be able to form connections with each other.

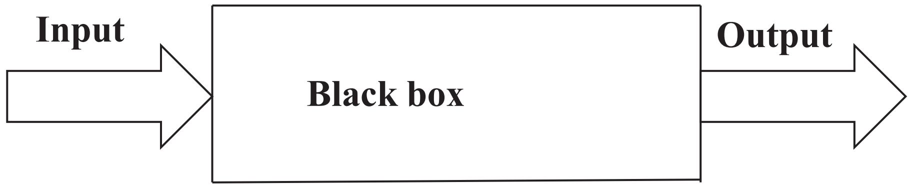
But this is only possible if two conditions are met.

First, the object must have the ability to *perceive* the impact of other similar objects (this can be information, information, data, signals, etc.). Secondly, he himself must have the ability to “generate” such influences that will affect other objects similar to him.

Thus, we come to the definition: a fragment of a system that is considered as a whole and is characterized only by its “input” (possessing, therefore, the ability to perceive influences from other fragments of the system) and “output” (by which it interacts with other objects of the system, including “responding” to their impact on him), is called a *black box.*

The black box is perhaps the most powerful abstract concept that exists within the framework of cybernetics. It is due to its introduction that it becomes possible to build closed systems that simulate the object or process under study. The black box is a "measure of our ignorance" about the system under study.

As a rule, it is indicated as follows in the form of a rectangle into which the input arrows (in) the characteristics of the black box – the parameters that they *convert* to the output (out) characteristics of the black box – are indicated by incoming arrows.



So, to set the black box, it is necessary to set the correspondence "input parameters" to "output parameters". It should be remembered that the internal structure of such a box remains unknown to us: we do not know how it is arranged, we do not know how it functions, we do not know what states it can have and how the transition between its states takes place (even if they have it is). The only thing that can be said is only to build a model for describing the input characteristics of such an object (a set of classes of variables to which it "responds"), and correlate it (with certain relations) with a model of output characteristics of a black box (that is, with a set of classes of variables in within the framework of which his "answers" can be expressed).

In the general case, it is thus assumed that such an object – a black box – is integrated as an "active element" into a certain system. This is especially evident in the case of a graphical (for example, in the form of a block diagram) description of the system.

The data (characteristics, parameters, information, etc.) that characterizes the input is often referred to as black box *input signals*. The data (characteristics, parameters, information, etc.) that characterizes the output is often called the black box *output signals*. Such terminology came from technical systems, to which the concept of a black box was first applied.

3.3. An operator as a model for describing the concept of "input - output".

When moving to mathematical models, at the mathematical level of description, such a converter of variables from one set (input characteristics) to another (output characteristics) is modeled by an operator (function).

The mathematical definition of the operator is known: Let V and W be some sets (for example, vector spaces).

Operator A acting from V to W is a map of the form A: $V \to W$, which associates with each element x of the set V a certain element of the set W. Typically, the notation $y = A(x)$ or $y = Ax$.

is used for the operator.

Thus, a black box acts as an operator when:

1) The parameters that characterize the input of the black box can be grouped into a certain set V.

2) The parameters that characterize the output of the black box can be grouped into a certain set W.

3) A certain rule is specified (algorithm, method of conversion, calculation, etc.), which allows one to calculate the value of y from the set W of the output signals of the black box from a known input signal - the value of x from the set V.

In view of the foregoing, the black box acts as a model of the system under study. And in the operator with which it is modeled, and, in fact, lies the mathematical model of the element that makes up our system. For this reason, the mathematical description of the black box is usually moved to the last stages of the simulation.

An important class of operators are the so-called *linear operators*. Although today the field of activity in modeling real systems using linear operators is extremely limited, they nevertheless still act as a powerful tool for mathematical analysis of systems. As we already wrote, system models also constitute a hierarchical system of logically related terms and concepts. Therefore, quite often it turns out that a system that is described *nonlinearly* at a *certain level* of logical depth of understanding, at a higher level, can well be described within the framework of an already *linear* apparatus and *linear* operators. Examples of such descriptions will be given in subsequent sections.

However, let us return to linear operators and give their definition.

Operator A acting from *V* to *W* is called *linear* if, for any elements x_1 and x_2 from the set *V* and any number μ the relations are satisfied:

1) $A(x_1 + x_2) = A(x_1) + A(x_2)$ (operator additivity property), and

2) $A(\mu\, x) = \mu\, A(x)$ (the property of homogeneity of the operator).

Examples of linear operators.

Here are a few examples of mathematical objects that are linear operators.

Matrix as a linear operator.

An ordinary matrix is a linear operator, if we consider it as a transformation of one column vector *x* into another column vector, *y*.

$$y = Ax, x = \begin{vmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{vmatrix}, y = \begin{vmatrix} y_1 \\ y_2 \\ \cdots \\ y_m \end{vmatrix}, A = \begin{bmatrix} a_1 & \cdots & a_n \\ \vdots & \ddots & \vdots \\ a_m & \cdots & a_{mn} \end{bmatrix}.$$

This relation is written for the case of a square matrix of the operator *A* of dimension m×n, which corresponds to the fact that the set *X* is a collection of column vectors of dimension *n* and *Y* is a collection of column vectors of dimension *m*.

Thus, the matrix, known from the course of higher mathematics, within the framework of economic cybernetics can be considered as a linear operator that models a number of properties of the black box. In particular, in this way, control models can be described – then the column vector x represents the information necessary for the solution, and the column vector y describes the solution itself. Matrix *A* in this case is an abbreviated record of the decision-making algorithm that corresponds to our model.

Differentiation operation as a linear operator.

The operation of differentiation – taking the derivative of a certain function – is also a linear operator.

In this case, X is the set of all (differentiable desired number of times!) functions, and Y is also the set of functions (but already differentiable the number of times, one less than the functions from the set X!).

Denoting the element of the set X by $f(t)$, and the operator A using $\frac{d}{dt}$, we easily verify the fulfillment of conditions 1) and 2) from the definition of a linear operator.

$$\frac{d\big(f_1(t) + f_2(t)\big)}{dt} = \frac{df_1(t)}{dt} + \frac{df_2(t)}{dt},$$

$$\frac{d\mu f(t)}{dt} = \mu \frac{df(t)}{dt}.$$

Note that, as is easily proved in the same way, the operator

$$A = Q(t)\frac{d}{dt},$$

Where $Q(t)$ – an arbitrary function is also linear. We emphasize that in the latter case, differentiation is always the first to act on the function $f(t)$, and only then is the multiplication of the result of differentiation by the function $Q(t)$. What is easy to see by comparing the results of two different action algorithms: the first – “differentiate first, and then multiply”, and the second – “multiply first, and then differentiate”!

Integration operation.

With the integration operation, after all of the above, no questions arise: of course, it – taking an indefinite integral – is also a linear operator. Actually, this was mentioned even in the framework of the course of higher mathematics – but then you did not even suspect that this, in fact, is a story about mathematical models!

The property of a certain object to be "differentiating" (sometimes – "difference") or "integrating" (sometimes – "summing up") is often set as the functions of some black boxes that perform certain control functions in the system. This is especially evident for radio circuits – however, economic systems also show us many similar examples. What is a bank? – This is an "integrating" object. What are ratings – economic or social? Is a differentiation procedure.

In fact, many more important mathematical details from the described example were left unanswered. But let us ask ourselves the question: do we really need these mathematical details, are they really so important for us in the process of solving practical problems ?! Having received a solution, we can always check whether it satisfies our equation and our boundary conditions! And mathematical rigor is not always needed when solving practical problems. Therefore – boldly introduce new mathematical operations, terms and concepts, being guided by only one, but the main criterion: *they should help solve the investigated problem!* Actually, this is exactly what scientists did throughout the development of science. For example, Paul Dirac – introduced a function that was very different from everything that was known to mathematicians before: they explained this only 20 years later. Richard Feynman introduced the mathematical operations that mathematicians still suffer from! Werner Heisenberg introduced operations that helped him explain quantum effects – and only then did mathematicians realize that these were well-known matrices!

3.4. Markov processes.

Consider the system. Let it be in a number of different states. Suppose, due to some reason – whether internal or external origin – the system will move from one state to another. Such transitions can be of two kinds. Transitions of the first kind – when the system passes from state i to state k: $i \rightarrow k$, and moreover, such a transition is *always* carried out. Thus, the processes in the system – for this class of cases – can be defined as a chain of successive states.

But there may be another case: the system makes transition $i \rightarrow k$ in a *probabilistic* sense. In other words, the final state of the system is no longer fixed (as it was before!), and for the *next* state of the system, in general, all states are open (including probability – the ability to stay the same).

For practical applications, the case when the probabilities of a system transitioning to another state depends *only on its current state*, that is, on the state in which it is currently located, but not on the state it was in before, is very important.

It is such cases that occur in many economic situations. For example, we are completely indifferent to what achievements the company had before: we, as investors, are interested in its forecast for the future – and it is determined only by its present position in the market.

Thus, *random processes* can serve as a sufficiently powerful apparatus for modeling dynamics, changing states and development prospects in economic systems. There are various ways to deal with such an accident.

For example, randomness can be "introduced" at the model level of the system under study by the fact that transitions between the states of the system are carried out at random times. Or – the transitions themselves are random - for example, there is a probability of a transition to several different states. In the general case, there can be everything: random moments of time, and random transitions between states, and the probabilities of such transitions themselves can be random - for example, when they occur under the influence of random changes in the environment external to the system under study. Note that in the latter case we come to a model for describing the *interaction* of the system under study with the external environment!

Of course, not all interesting cases, from the point of view of a specialist in the field of economics, have a well-developed mathematical apparatus. However, a class of random processes is distinguished for which very powerful mathematical results were obtained, which allows them to be successfully applied in many fields (see, for example, the next section 3.5).

A random process is *Markovian* when any *additional* information, except knowledge of its current state, is not essential for predicting a further change in the state of the system.

It is the requirement of the future that depends only on the present and leads to what Markov processes are often called "processes without memory".

There are a fairly large number of options for the mathematical apparatus for Markov processes. But this will be discussed in special disciplines at senior courses.

A specialist-economist very often in his practice encounters a situation when, in order to construct a mathematical model of an economic object, he has to turn to those branches of mathematics that are completely new to him. And then he opens the mathematical books, and begins to understand the mathematical apparatus necessary for him. At the same time, he does not need to get to know him in great detail: it is enough when he, firstly, understands the provisions laid down in the foundation of a particular mathematical theory or concept, secondly, he is convinced that these provisions do not contradict the provisions of his model (if there is such a contradiction – you have to abandon either mathematics or the model!), and thirdly, when he learns to use this mathematical apparatus - that is, when he learns to solve problems using it.

3.5. The concept of "feedback".

So, we can consider the management of the system as the implementation of *transitions* between its states. But what is a *condition*? This, by definition, is something *stable*, that is, it means that the parameters that characterize it take on some stationary, time-invariant values. However, each system is exposed to *random (or deliberate) environmental influences*. In this case, we expect that the system, being deduced from a certain (for example, equilibrium or stationary) state, has the ability to "spontaneously" return to *stationary* characteristics, that is, to the state in question.

But what if the system does not have this property? To answer this question, we ask the counter: how then can we talk about the "state"? Thus, we conclude that talking about the state for the system makes sense only if this state is *stable*. In other words, we expect that the state of the system should have certain stability boundaries, that is, with a small change in its parameters (still remaining *inside* the *stability boundary*), the system will *remain* in the same state.

But there may be situations when such stability limits for the system are “too narrow” and we would very much like to *expand* them. Is it possible to do this, and if so, how?

In order to answer this question, you need to introduce the concept of "control action". In order to describe it, we will split the entire set of parameters characterizing the system into two classes. First class: parameters that we cannot influence. The second class is the parameters that we can - within certain limits - change. These are the parameters that we can change, and are called control.

Control parameters are the characteristics of the system that have two properties: firstly, they can be changed in the direction we need (for example, in magnitude and sign) by external influences with respect to the system under study, and secondly, they determine the boundaries of stability system (in particular, the rate at which other characteristics of the system tend toward its stationary value, which characterizes a given *state of the system*). In general, the control parameters will be different for each state.

Note that we do not associate control parameters solely with the process of *returning* the system to this state. The definition is given in such a way that it also allows controlling the *transition* of the system to a new state. For example, this is possible when the boundaries of system stability (in a given state!) Under the influence of external control are *narrowed* to a value when external random influences (factors) already lead the system beyond these boundaries.

As follows from the above note, we can, in the general case, divide all methods of controlling the system into two alternative classes.

- Management designed to ensure the *stability of the system in this state.* This is ensured by the so-called *negative feedback*.
- Management designed to ensure the *transfer of the system from one state to another.* This is achieved through *positive feedback.*

How is *feedback* generally organized?

Imagine a system. Let her *deviate* from her current state. We can judge this by *changing* the values of a number of parameters that characterize it. Now we are forced to make a decision – that is, to determine the *goal* of our management: whether to facilitate the *return* of the system to its original state (negative feedback - the adjective "negative" has not only literal meaning (we will see below!), But also emphasizes that we strive *reduce* the changes made by the environment), or vice versa, *increase* this deviation so that the system moves to a *new state* (positive feedback: again, the adjective "positive" has not only *literal* meaning, but also symbolic, emphasizing our desire to *increase* the deviations that occur in the system).

In fact, the *positive and negative* feedbacks thus form a *control loop*, which has a closed form due to the appearance of the possibility of dosing control actions and analysis of their results.

There are only two known – and complementary! – a way of managing people in social and economic structures: these are the methods of "carrot" and "stick". In fact, they are often transformed into methods of *encouragement and punishment*. This is the implementation of the same positive or negative feedback. Encouragement corresponds to negative feedback that captures certain actions of the employee, that is, *encouraging* him to continue his current activities. Now we can say – encouragement stimulates his being in this state. Punishment is the opposite: encourages him to *change* his current state to another. By the way: now it's also become clear to us that the reasons for the existence of precisely *these two systems* for managing an individual are an optimal set that allows for the effective management of their activities and behavior. Using *only one* of these methods, therefore, means *inefficiency* in management. In an efficiently operating company, both ways of rewarding and ways of punishing employees should be clearly fixed. Of course, at the same time, rewards and punishments should relate *exclusively to that area,* the results of which depend on the *personal activity* of the employee, that is, they are determined by the *state in which the employee is located*. Misunderstanding of this circumstance – when an employee is punished or rewarded not for his personal actions, leads to inefficiency in managing economic systems. at the same time, rewards and punishments should relate exclusively to that area, the results of which depend on the personal activities of the employee, that is, they are determined by the state in which the employee is located. Misunderstanding of this circumstance – when an employee is punished or rewarded not for his personal actions, leads to inefficiency in managing economic systems. at the same time, rewards and punishments should relate exclusively to that area, the results of which depend on the personal activities of the employee, that is, they are determined by the state in which the employee is located. We now turn to the mathematical form of the description of the above.

The linear case – the Malthus model.

Consider a system that is characterized by only one *parameter x* – such systems are called *single-component*. Let the state of the system be characterized by its value x_0. Due to external influences, the system changed and became characterized by the value of the parameter x_1. The simplest case of control is when we realized such conditions that the rate of change of the parameter is proportional to its deviation from its equilibrium (stationary) value x_0.

Mathematically, this can be written as follows:

$$\frac{dx}{dt} = \gamma(x - x_0) \; or \; \frac{d\Delta x}{dt} = \gamma\Delta x, where \; \Delta x = x - x_0.$$

It is more convenient to write this equation immediately with respect to a change in characteristic – that is, a change in a system parameter Δx.

In the equation γ is so-called control parameter, which also characterizes the system, and which we can change both in magnitude and sign – for example, make it either positive or negative – with the help of external (controlled) influences.

The solution to the equation is written as

$$\Delta x = (x_1 - x_0)e^{\gamma t} \text{ либо } x = x_0 + (x_1 - x_0)exp(\gamma t).$$

Here x_1 – the value of the characteristic *x* (deviation from equilibrium) at *t* = 0, that is, at the initial moment of time.

The solution shows that for $\gamma > 0$ the system will increasingly *move away* from its *equilibrium* state characterized by value x_0. On the contrary, with $\gamma < 0$

the system will *return* to its *equilibrium* state. Thus in the first case, when $\gamma > 0$ – there is a *positive* feedback, and when $\gamma > 0$ – *negative* feedback.

The speed with which this removal/coming closer will be carried out depends on the absolute value of the control parameter – from $|\gamma|$. The larger this value, the faster the system moves away/returns to an equilibrium state.

So, within the framework of this mathematical model, we get the opportunity to regulate – that is, control the system by:

1. Creating positive / negative feedback.

2. Changes in the *strength* of this feedback (modulus of the control parameter) $|\gamma|$.

An example of the considered equation describing the *real* socio-economic situation is the so-called *Malthus model* for the population. It is based on the "simple and natural" assumption: the increase in the number of people is proportional to their available number. In this case, the equation describing the deviation of the system from a certain *initial* number of people N_0 can be written as follows: $dN/dt = \gamma N$. Of course, here $\gamma > 0$, in order to take place is an *increase* rather than a decrease in the population. The solution to this equation has the form $N(t)=N_0\exp(\gamma t)$, – it is believed that at $t = 0$ the population became N_0. As can be seen from the solution, the population in this model is growing rapidly, - the period of doubling the population can be calculated by the formula $T = \ln 2/\gamma$: according to demographic statistics, this period of time today is 40 years. In the Malthus model, we obtained a population growth *exponentially*. At the same time, it is known that the resources that a country has (and the planet as a whole!) Increase in *arithmetic* progression. But then we come to the conclusion that, as time passes, population growth occurs *faster* than the growth of resources! In other words, the *relative* amount of resources – the amount of resources per person – will *decrease* over time. This, in fact, is the conclusion of the Malthusian theory. He made this conclusion at the beginning of the 19th century, and at the end of the 20th century, scientists who formed an informal organization called the Club of

Rome came to the same conclusion. Of course, they came to him, using much more sophisticated theoretical and mathematical models. Actually, it is in such a simple model, which turned out to be surprisingly little sensitive to subsequent refinements, that are the reasons for all the increasing calls for birth control (that is, to reduce the control parameter γ). Of course, this will not solve the problem – but at least it will give time for decision-making. Maybe achieve that $\gamma > 0$ – at least on a global scale? However, as is easy to see, this value is *unstable*: justγ will become positive – population growth will begin again, and as soon as it becomes negative – the population will begin to decrease. Of course, this will not happen immediately – but such a management organization already affects the *entire population of the Earth*, and therefore requires *completely new methods of management and coordination on a global scale*. It is impossible to do this today. So what to do ?! First of all, to *study* this problem, build new models, consider new possible scenarios.

Nonlinear Feedback – Verhulst Model.

The case was described above when the system *deviated* from its original position and rapidly moved away from it, within the framework of this model, arbitrarily far. However, we expect – in any case, models of systems are built precisely on the basis of this – that, sooner or later, our system will enter a new state. In other words, now we are faced with the problem of a mathematical description of the *transition* of a system from one state to another.

To build such a model, we ask ourselves the following question: why is "braking" of a change in the system characteristic possible at all? For example, this can be done as follows: as soon as the value of the characteristic of the system x begins to approach the *desired new* value of x_2, the value of the control parameter γ should decrease and reach zero at $x = x_2$.

In other words, to describe the control of transferring the system to a new state, we must consider the case when there are dependencies of the control parameter on the current characteristics of the system. As a rule, we obtain nonlinear differential equations.

For example, for our equation, its modification looks like this:

$$\frac{dx}{dt} = \gamma(x)x.$$

The simplest case is when $\gamma(x) = 1 - x$, and we get an equation called the Verhulst equation or the *logistic* equation (any linear dependence of the control parameter on the characteristics of the system can be brought to this form by coordinate transformation) $\gamma(x) = 1 - x$

$$\frac{dx}{dt} = (1 - x)x.$$

It is easy to see that this equation describes the transition of the system from the "unstable" state $x = 0$ to the stable state $x = 1$ – only positive x values are considered.

In the most general case, *deviations* from equilibrium – that is, from a *steady* state – are most often described in the framework of a *linear* approach. Even if *nonlinear* additives are considered, they are assumed, in a certain sense, to be "small" in comparison with linear terms. Therefore, we can conclude: for control through *negative* feedback, as a rule, a *linear* description is sufficient. Since linear methods in mathematics are well developed, it is therefore not surprising that the main successes in cybernetics (especially technical cybernetics) have been achieved precisely in the field of systems control in order to *maintain* their current state. At the same time, in the field of *economic* cybernetics, a huge number of tasks are completely opposite in nature: it is necessary to *control the process of transferring* the system under study to the state that we need. Therefore, *nonlinear* mathematical models are the main object of study in economic cybernetics. The mathematical apparatus for their study is very complicated, for this reason there are not many results achieved. However, in the framework of technical cybernetics for nonlinear problems, results are also achieved, very few.

The solution of the Verhulst equation can be written as

$$x(t) = \frac{x_0 e^t}{1 + x_0 e^t}.$$

Here through x_0 the value of the characteristics of the system at the initial moment of time is indicated, at $t = 0$. From the solution it follows that at $t \to \infty$, $x(t) \to 1$.

The question arises: can we talk in this case about the presence of *feedback* in general? Maybe it would be more correct to talk about a system *model*? Much depends on what problem we solve, that is, on the *purpose* of our research. As a rule, the question of building a system model is nothing more than a *stage* in the preparation and selection of a management system for a given social or economic object. This idea will become more understandable when the Verhulst equation is written in *dimensional* form, that is, as it is usually obtained in modeling:

$$\frac{dx}{dt} = ax - bx^2 = ax\left(1 - \frac{b}{a}x\right).$$

In this form of writing, *control parameters* a and b are explicitly entered through the change of which we can control both the *final* state of the system and the process of its achievement.

Interpretation and generalization of the Verhulst model: "Catch quota" as a model of optimal management.

The Verhulst model appeared as the simplest generalization of the Malthus model to the presence of "natural restrictions" on fertility, leading to the death of individuals. This model often describes the reproduction of biological objects of various kinds – from bacteria to higher organisms – such as fish.

In connection with the latter, let us consider, using the example of fish, the *organization of a control system* for their abundance taking into account catch. Such a task reflects our natural desire to use a resource – in this case, fish – for our needs. At the same time, however, we want to *control the number of fish* in such a way as to achieve the maximum possible catch without the fish disappearing. Thus, we will consider the problem of the *optimal* use of natural resources. Moreover, the term *"optimality"* means that 1) fish should be caught as much as possible, but 2) the resource should not be depleted.

Since our intervention is external to the system, the Verhulst equation $\frac{dx}{dt} = (1 - x)x$ need to modify.

Consider the two simplest options for modification.

First of all, we can catch fish at a *constant speed* indicated β (the number of fish caught per unit time, for example, annually). In this case, the equation Verhulst will take the form

$$\frac{dx}{dt} = (1 - x)x - \beta = -\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{4} - \beta\right).$$

From this equation it follows that for $\beta > \frac{1}{4}$ the number of fish can only decrease, because in this case the derivative will always be negative. In other words, if we catch *annually* (as a natural time interval it is convenient to choose 1 year – the time of the reproductive cycle of fish) more than 25% of the *stationary* number of fish (that is, those that would be without a catch – in our notation their number is 1), then the fish resource will be depleted, that is, the number of fish will rush to zero. When $0 > \beta < \frac{1}{4}$ – the fish resource is established at a certain level, which makes up some part of the maximum possible $x = 1$. At the same time, however, the *maximum catch quota* $\beta = 1/4$ is unstable (any arbitrarily small excess will lead to the disappearance of the fish system), and therefore must be declared *inadmissible*.

Maybe try to organize fishing in a different way? For example, we will set the catch quota as a value proportional to the number of fish already available? Then we get the equation

$$\frac{dx}{dt} = (1 - x)x - px = (1 - p)x - x^2.$$

Here, the px value sets the speed of fishing β. From the obtained equation, it is obvious that the inequalities $0 < p < 1$ hold. Under these conditions, the *stationary* number of fish is set at $x = B$, where B is found as a solution to the equation

$(1-x)x = px$. The catch rate can then be calculated using the formula $\beta = pB$. We ask ourselves a question: when can this speed be *maximum*? The answer to this question is easiest to find from geometric considerations. Point B is located as the *intersection* of the graph of a quadratic parabola $(1-x)x$ and a straight line px. Highest catch rate $\beta = px$ equal to the largest ordinate $(1-x)x$ of the function graph, and this is achieved at $x = 1/2$. In this case, the value of $p = 1/2$ (it is necessary that the value of px be equal to 1/4 – the maximum value of the function $(1-x)x$, which is achieved at $x = 1/2$. And this is achieved, in turn, at $p = 1/2$.

Thus, for our task, the maximum fishing speed is set at $\beta = 1/4$, but now, as is easy to see from the last equation, a *stable* amount of fish is established.

So we gave an example of a situation where consideration of *different* scenarios of system management – in our case, these were different scenarios of fish catching – allows us to achieve a *stable* transfer of the system to a new state. Of course, problems that are important for practice will most likely not have such a simple form – however, the *general methodology* for solving them will be the same: first we select the *appropriate model of the system* and formulate a basic mathematical model for it. And then – we will explore different *control methods* that can be implemented within the framework of this model. Often, for this we have to clearly highlight the assumptions that were the basis of the basic model, and examine whether we can abandon them – and how the mathematical model of both the system and the management of this system will change.

3.6. Operational project chart to study and improve the management system.

The entire range of work on the study and improvement of the management system to achieve this goal is carried out by the operational group of system analysts or is also called the *operating side* (OS). This set of work is carried out in the interests of the decision maker (DM). The decision maker may reject the project, but may accept. In fig. 5 shows an exemplary diagram of the stages of an operational project for the study and improvement of the management system. We give a brief description of the stages presented in Fig. 5 operational research.

1. In the most general case, the cause for the study and improvement of the system are fixed symptoms that reveal problematic issues in the system.

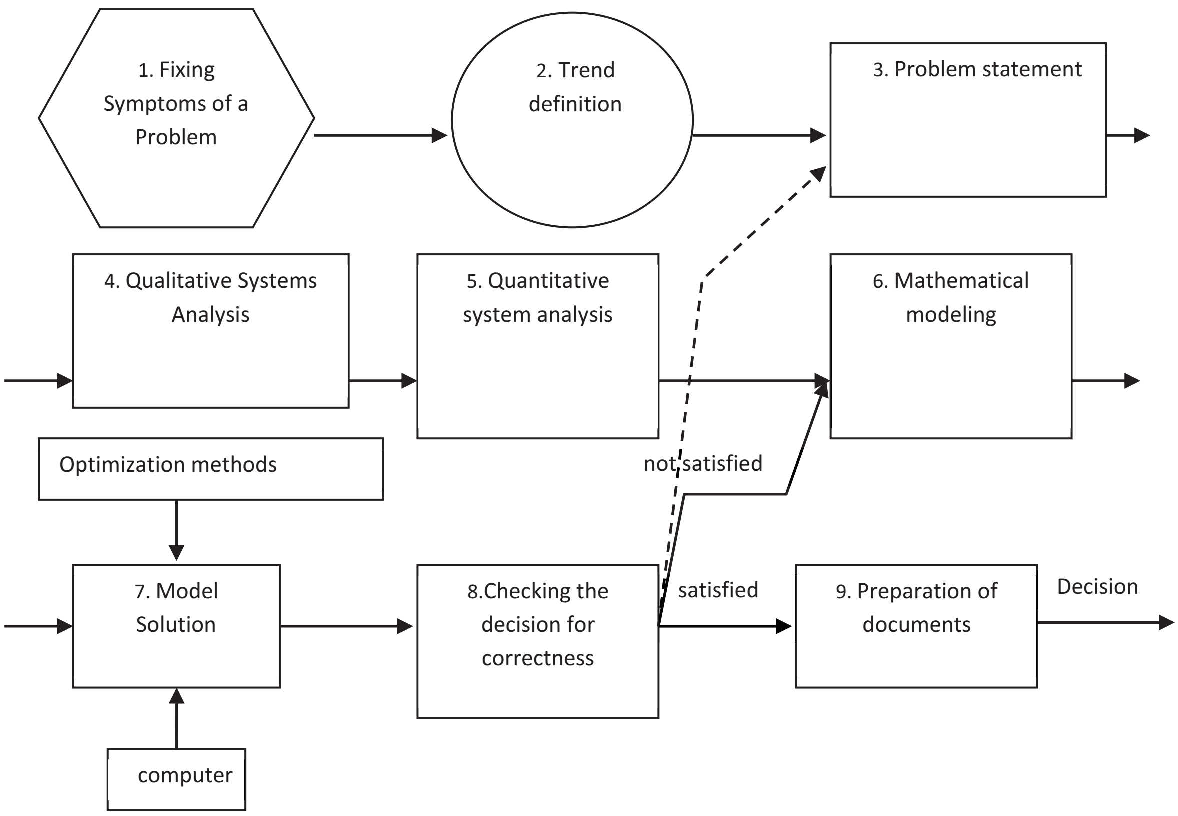


Fig.5. The stages of the operational project to study and improve the management system

2. The identified symptoms of the problem can form a connected chain of facts (tendency) that helps to formulate the problem.

3. The most important stage in the study of the system is a clear statement of the problem, which is present at this level of the life of the system.

4. Qualitative system analysis is the splitting of a holistic system into separate elements (entities). To do this, you need:

- isolate the system under study from the parent system, i.e. establish the boundaries of the system to be optimized. Extending the boundaries of the system increases the dimension and complexity of a multicomponent system, thereby complicating its analysis,

- formulate the goal carried out by the system,
- list the factors that affect the achievement of the goal,
- identify possible limitations within which the system can be improved.

5. Quantitative system analysis involves describing all of the factors that are involved in the operation at a quantitative level, that is, on the basis of measurable parameters. For this:

- establishes a *criterion* (*performance indicator*) (*K*) – a quantity that quantitatively measures the degree of achievement of a system's goal and allows comparison of different decisions on efficiency. In case there is a single-purpose decision-making, then *K* is a scalar; if multi-purpose decision-making, then *K* is a vector;
- quantitative internal parameters of the system are introduced that measure the factors involved in the description of the system. The whole set of these factors must be divided into two parts:

a) uncontrollable (uncontrolled) factors that the OS does not control and cannot change in this particular system (operation). Parameters $a_1, a_2, \dots, a_k$ we will consider these factors as the coordinates of the vector, and denote it $a = (a_1, a_2, \dots, a_k)$. Uncontrollable factors $a_1, a_2, \dots, a_k$, based on their awareness, they can be divided into three groups: fixed, random, indefinite.

Fixed uncontrollable factors – are factors whose values are precisely known by the OS. *Random uncontrolled factors* are random variables whose distribution laws (functions) are precisely known by the OS. Uncertain uncontrolled factors are deterministic or random variables, relative to which the OS is known only to the region of possible values or the class of possible distribution laws,

b) *controlled parameters (variables)* $x_1, x_2, \dots, x_n$ – quantities that the OS can control and change them at its discretion. We will consider $x_1, x_2, \dots, x_n$ – as the coordinates of the vector – a single element of the mathematical model, call *strategy* and denote $x = x_1, x_2, \ldots, x_n$.

6. The essence of mathematical modeling is the establishment of quantitative relationships between the entered values of K, a, and x in the form of the so-called *operational (mathematical) model*. First, we reveal the contents of modeling in the general sense, and then in the formalized (mathematical).

Modeling – this is a research method (knowledge) the world around us, consisting in replacing real objects or phenomena with their deliberately simplified images (models) in order to study these images and the subsequent transfer of the results and conclusions to objects and phenomena of the real world.

The relationship (relation) between an object of the real world and its model can be illustrated graphically with the help of an enlarged modeling cycle (Fig. 6).

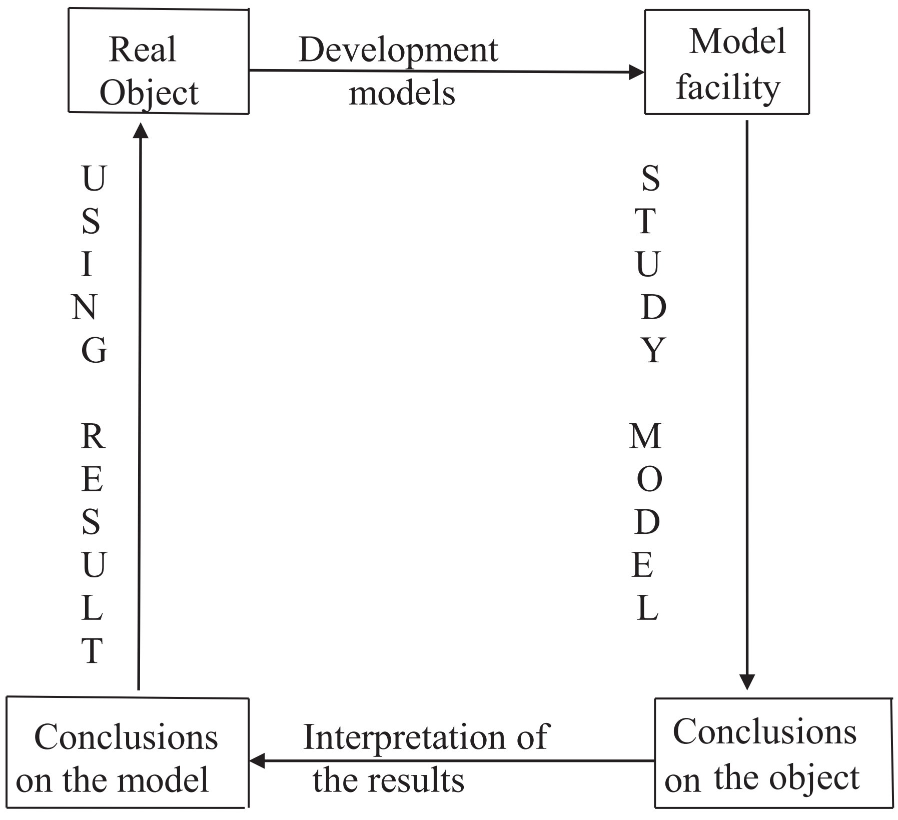


Fig.6. Integrated Simulation Cycle

Model (in science) is a substitute object of the original object, a tool for cognition that the researcher puts between himself and the original object and with the help of which he studies some of the properties of the original. The model is another material or mentally presented object that replaces the original object in the research process. Correspondence of the model properties to the original object is characterized by ***adequacy*** – the information obtained during the study of the model can be transferred to an object with one degree or another degree of reliability

Mathematical modeling is a means of studying a real object, process or system by replacing them with a mathematical model, more convenient for experimental research using various mathematical methods using a computer.

Mathematical model is the image of the object under study, created by the researcher (subject) using a certain formal (mathematical) systems to study (assessments) certain properties (or functioning) this object.

When building a mathematical model, you must be guided by the following basic principles:

1). It is necessary to measure the accuracy and detail of the model, firstly, with the accuracy of the initial data available to the researcher, and secondly, with the results that you want to obtain.

2). The mathematical model should reflect the essential features of the studied object or phenomenon and at the same time should not greatly simplify it.

3). A mathematical model cannot be fully adequate to an object or phenomenon, therefore, for its study it is better to use several models, for the construction of which different mathematical methods are used. If similar results are obtained, then the study ends. If the results are very different, then the statement of the problem should be reviewed.

4). Any complex system is always subject to small external and internal influences (disturbances), therefore, the mathematical model must be stable, i.e. maintain their properties and structure under these influences.

Elements of the model contain all the information used in the calculation of the project. The process of constructing a model is very time-consuming and requires a clear understanding of the specific features of the system under consideration. There are many different classifications of mathematical models. In particular, there are models multi-criteria dynamic, in which, in addition to the parameters a and x, the time variable is clearly present, and static, in which this variable is not. In reality, all processes occur in time, so dynamic models, generally speaking, more accurately describe reality. However, it should be borne in mind that all these models are based on the foundation of single-criterion optimization method and limited to simpler static models without a clear understanding of which it is impossible to work with a more complex mathematical apparatus. Moreover, the performance criterion is presented as a numerical function of only strategies and uncontrollable factors, i.e., $K = f(x, a)$, where f is the model of the objective function, it establishes the functional dependence of the criterion K on uncontrolled parameters a and controlled values of x.

For the objective function, the direction of improvement of the criterion is indicated

$$K = f(x, a) \rightarrow \min (\max). \tag{3.1}$$

This expression determines the meaning of system optimization, that is, it is the mathematical equivalent of the purpose of the operation. Setting the objective function is the first part of building the operating model.

The second part of the operating model is a mathematical description of the restrictions on the choice of variables x. All restrictions in general form can be written in the form of inequalities and equalities:

$$\varphi_i(x, a)\ (\leq,, =, \geq)\ 0,\ i = 1,2, \ldots, m = \overline{1, m}. \tag{3.2}$$

Each function $\varphi_i\,(x,\ a)$ is called a *restriction function*. In some problems, there are requirements for the form of the variables x or K.

$$\left.\begin{matrix} x \in D \\ K \in M \end{matrix}\right\}. \tag{3.3}$$

For example, the requirement often arises that x or K must be integers. In some cases, they must belong to some standard set of values.

A model in the form of (3.1), (2.2), (3.3) is a *model of an operational type* or an *optimization model* (not an optimization one – without an objective function).

The optimization model (3.1), (2.2), (3.3) allows us to pose the problem of *optimizing* the system (operation) as a mathematical problem: find controlled variables x^* that satisfy the constraint system (4.2), (4.3) and provide the best value for the criterion K the objective function.

7. The solution of the mathematical problem posed requires the use of optimization methods, including, in addition to classical mathematical methods, also special methods of operations research associated, for example, with multicriteria optimization methods or when the model contains random uncontrolled parameters. It should also be noted that for the most part, real optimization problems lead to a large amount of calculations (up to tens of thousands). Therefore, modern methods for finding optimal solutions are focused on the use of computer tools.

8. Comparing the obtained solution with a meaningful (verbal) statement of the problem, one can detect contradictions or some incorrect elements of the solution. Errors in the mathematical model or failure to take into account some significant limitations may be the cause of the incorrectness. At this stage, the decision maker may be involved. If the solution obtained is acceptable - it is accepted, if not - you must return to the stage of mathematical modeling or even to the earlier stages of the study.

9. The found optimal solution x^* allows you to prepare a control solution in the form of a document for the decision maker.

Thus, operational research is an iterative process that converges to a certain optimal solution. The considered scheme is approximate. It allows you to better understand the meaning of system analysis as a science - on the quantitative justification of optimal solutions based on the construction and use of a mathematical model. Unfortunately, the process of translating into a quantitative description of complex systems is not a technology. The mathematical model may turn out to be successful or unsuccessful for practical solutions. In addition, far from always the whole complex of goals and objectives facing the modeled object can be expressed in the form of some objective function.

To illustrate the main stages of the operational project and the constituent elements of mathematical models by examples of constructing economic and mathematical models and their optimization for some substantive problems, topic 5 is devoted.

3.7. Information security of computer control systems.

Companies that are pioneers in the use of information technology, not only receive significant benefits, but also carry increased risks. One of the most pressing issues of our time is the issue of ensuring information security, both of various government agencies, as well as commercial organizations and personal data. “Recently, more and more information, including critical information for individuals, organizations or states, is stored, processed and transmitted using automated information processing systems (AS). "An information processing system is a combination of hardware and software, as well as information processing methods and personnel actions necessary to perform automated information processing."

Even a decade and a half ago, when the information economy was at the initial stages of implementation in our lives, according to the research of information security experts, by that time, a skew of concepts in the field of data protection was becoming more and more noticeable. Speaking about the economic feasibility of measures to support for information security, many primarily mean protection against viruses and hackers, but according to reports from leading organizations over the past ten years, the actions of insiders (company employees) have caused the most damage, as shown, for example, in the FBI's annual reports Computer Crime and Security Survey According to these studies, the estimated damage from careless and unlawful actions of employees is several times greater than the amount of damage caused by the actions of viruses and hacker attacks, or by attempts of malicious direct implementation in order to extract information.

And this was back in 2007, when the number of incidents caused by external and internal troublemakers was comparable.

This result was quite natural. Despite the fact that there are actually many more external intruders, but, firstly, they are less motivated, discarding a small number of hired professionals, we will get a huge mass of schoolchildren and students who just out of curiosity try the downloaded utilities without any specific goals and sometimes without even knowing what to do with the information received. Secondly, they are confronted by powerful and mature technologies of perimeter protection, that is, an external attacker needs great qualifications to overcome all these barriers.

Nowadays, even despite a significant improvement in breakthrough technologies through network and information security, the situation has not changed much. Most often, it's the strategically important areas or the largest enterprises that can either be inflicted irreparable damage by violating their information security, stealing data or funds, in the case of a financial organization, from external attacks. As for small and medium-sized businesses, it is still rarely hit, because it is not of particular interest, even considering the much weaker means of protection against outside interference.

The internal intruder, especially if his actions are conscious, and not a mistake, may have more incentives. From a banal resentment to material gain in the case of bribing by competitors, and the possibilities are much greater, since it is already legally legal network user, has access, including to the confidential resources of the organization, can use corporate applications and the data processed by them legally. In addition, in about 40% of cases of external interference in the work of the enterprise, there is no circumvention of the information security system, because it used the internal resources of the enterprise and its interested staff members to strike. But if this is so, why are great efforts spent specifically on protection from external threats? There are several reasons for this.

For starters, building a defense system against an external enemy is almost always a much simpler task. This is a well-known and already well-trodden path, as any of the information security personnel, even with a low level of skill, is ready to start listing the necessary means of protection against threats, because in most cases they are similar, and the details require adaptability to a specific situation already in place, when an attempt occurs interventions. In addition, while constructing the external line of defense against external threats, no one affects the performance of the information system itself. All business applications work fine, the price of an administration error is, by and large, only a short-term lack of Internet access or minor disruptions in the operation of the internal network of resources related to quick-fix problems.

Protection from the internal enemy is much more complicated and requires a lot of effort, as well as costs. It consists of ensuring the security of the applications themselves and competent administration, which, first of all, implies the existence of clear privileges for company employees to access the resources of the information system, which in a worded form are called internal security policy. These privileges should be sufficient to ensure normal operation and at the same time minimal in terms of access and the ability to manipulate information. Such internal access control systems are currently used by many information security service providers, having been copied from the state system of access levels to classified information, only projected already on the internal structure of the company. And often with the appearance of such a problem, more problems are seen than solutions.

You can list them for a long time, since the main problems cling to each other. For example, the insecurity of a number of applications forces us to use additional security tools, however, these tools need not only to be acquired and properly implemented, but also to be maintained on an ongoing basis for each individual device used on a computer network. And if usually there are no problems with the implementation process, since either full-time specialists or hired consulting companies cope with it, the difficulties of maintenance appear later in the system administration process. After all, the management of accompanied protective equipment is often carried out separately from those already used in the company, including standard mechanisms. And this means that sooner or later, depending on the scale of the information system, there comes a moment,

The discrepancy also occurs because there are no procedures governing the introduction of changes both in the information system itself and in the settings of the security mechanisms controlled by it. And, actually, the very introduction of changes to the real system settings is much simpler and faster than the regular execution thereof with documentation of the main amendments made. Yes, and dialing an administrator number or running to him along the way is easier than writing an application. As a result, at a certain point in time, it is practically impossible to recreate the real picture of what is happening, as well as it is impossible to answer the question - why do these users or groups of users who do not have sufficient privileges for such a resource have access. It is in this way that the history of all updates made is lost,

The price of an error for such improper administration is measured either by providing the user with unreasonably large competencies, that is, with certain precedents, this will be fraught with the creation of a huge vulnerability in the information system, or with the restriction of the access necessary at some point, acceptable for his privilege level, while perhaps the organization’s tasks are disrupted.

Formally, there are several ways to solve these problems, such as the introduction of a full-fledged centralized management system and the introduction of a system for accounting for settings and changes in the settings of the information system or any of the segments of the company's IT sphere, even outside the jurisdiction of the information security department. However, the universal management console for all applications does not save, since such a solution does not answer the question - on what basis, who and how should make decisions about what changes are needed. To streamline the administration activities at the enterprise, full-fledged regulations and other documents should be developed that describe the rules of work and interaction of all subjects of the information system.

But it's worth remembering that it is not possible to solve this problem only by organizational methods. This is due to such things as a banal lack and insufficient qualifications of administrators, overload of specialists and, most importantly, the lack of mechanisms to verify the actual situation. All this leads to the fact that even if there is some formal management system, control over the information system and data security issues in it is still lost without adequate management measures. Therefore, given such processes, it is worth noting that often a simple increase in the staff of IT units and information security units, even only at the expense of administrative specialists, can often exacerbate problems. In these structures, in turn, there are units specializing in individual subsystems,

Ensuring increased requirements for information security involves appropriate activities at all stages of the life cycle of information technology. The planning of these activities is carried out upon completion of the risk analysis phase and the selection of countermeasures. A mandatory component of these plans is the periodic verification of the compliance of the existing regime with the security policy, certification of the information system (technology) for compliance with the requirements of a specific security standard. All of the above and there is risk management.

By and large, to manage any complex system, it is necessary to create a strict but simple regulation of system maintenance and ensure that the system settings are changed in accordance with this regulation.

With regard to ensuring the security of the information system, this can be represented as follows. First, include in the document a document that should clearly describe who and on what basis should have access to the resources of the information system. In addition, it is necessary to have a single point of interaction between the organization's employees and the information system through which they can formulate their wishes for providing access to certain resources of the information system within the framework of the current information security standards of the enterprise. And to manage such processes, an enterprise needs to have flexible tools to control the correctness of information network settings. It is worth noting that in recent years several large corporations have already been engaged in such developments. Information security divisions of Oracle and IBM offer their solutions. A feature of the proposed solutions of this kind is that they combine technical and organizational approaches to safety management that do not work separately.

When implementing such systems, it is assumed that the organization already has a formulated security policy. This policy, together with information on IP, further serves as the foundation of the management system. As for the description of the information system itself, it is usually necessary to know the following:

- A list of information resources. A resource can be understood as specific servers and folders on them, operating applications, equipment, and even network segments.

- Responsible for the security of these resources. This may be the owners of resources, heads of departments, curators from the security service and others.

- Responsible for administering these resources.

- How information system resources are interconnected. Sometimes for the normal operation of the application a set of settings is required – from the settings of the application itself to the switching equipment. After all, even if we make all the settings, but forget to write the permission rule on the internal firewall, the solution to the whole problem will be disrupted.

- The regular structure of the company. What access, and to what resources does the employee holding a certain position have?

Thus, on the basis of all the above information received, the management system builds an ideal model of the enterprise's information network that complies with information security standards. This moment can be considered the starting point in the work of the security management system. From now on, all communication on changes in the settings of the information system begins to take place through a specialized document management system, which is part of the security management system, which is under the supervision of the information security department.

It is worth noting that the domestic system of integrated security management is distinguished from foreign analogues by a special translator that allows you to overcome the language barrier and provides an opportunity for everyone to work with terms that are clear to him. For example, if this is a company's management, then in the system it can use a set of functions for managing employees and positions, without unnecessary information add-ons, while information security and IT specialists will work with user accounts, their access rights, etc.

An application for changing access, compiled in the security management system, will be checked for consistency with the requirements of the security policy, agreed with the owners of the resources and sent to fulfill to administrators. To detect inconsistencies in the model of the information system and its current state, the security management system allows sensor agents. Such agents regularly monitor all the settings of the operating systems, applications, security tools, and network equipment related to the enterprise's network security.

By the mismatch of the security management system of an enterprise's information system, one should understand either the administrator's unfulfilled necessary actions to administer the information system or the actions performed by him bypassing the order adopted and approved by the organization. For example, granting unnecessary powers to any user or unlawful restriction of a user's rights.

Information about non-compliance immediately goes to the security services and to the IT service. After all, each of them is connected with the fact that one of the employees either acquires rights to access the resources of the information system or loses them. This means that he may receive excess information or lose access to the information he needs. And this, as already noted, is equally unacceptable, because it poses a threat to security or leads to a disruption in the performance of business tasks.

The presence in the document management system of the mechanism for archiving applications for changes in access to the information system will allow at any time to understand who has access to the resources of the information system and who requested the provision of this access. Using the described approach to managing information security is a serious change in the usual rhythm of the information system. But the effort expended is more than paid off. The benefits of implementing security management systems are clear. And above all, this is an increase in the security of the information system, since from now on all changes made to the settings will be monitored and made in strict accordance with the organization's information security policy. An additional bonus will be the reduction in costs associated with the management of document management.

In addition, after the introduction of such a management system, ensuring information security ceases to be the inheritance, responsibility and duty of only narrow specialists. In information system management the organization's management begins to really actively participate: after all, it is they who now formulate the requirements for settings through the application mechanism.

As for the information security market in our country, the situation here is as follows. In Ukraine, information security companies are most seriously concerned with companies related to information technology, the banking sector, mobile communications, and companies involved in securities transactions. As for other organizations, according to the results of various studies, the leaders of many of them are aware of the main types of threats, but do not pay due attention to these issues, believing that ensuring information security does not make sense if there is no visible threat. That is, the main problem in the field of information protection is the insufficient attention paid to it by company management and, as a consequence, the lack of financing.

Nevertheless, gradually the leaders of Ukrainian companies are changing their view of information security, starting to treat it as one of the ways to increase the competitiveness of the company.

Turning to the structure of the applied systems for ensuring integrated information security, it is worth saying that, despite significant progress - at present, the lion's share of the domestic information security market is still composed of firewalls, Intrusion Detection Systems (IDS) and anti-virus systems. However, these funds for the most part have ceased to meet modern requirements, within the framework of the current realities of information security presented to protective systems.

This is understandable, since the time intervals between the appearance of a message about another new point of vulnerability in software, the release of a patch and the creation of a program that uses this vulnerability are being reduced very quickly today. IDS only detect computer attacks. A parallel can be drawn between such a system and a thermometer: the latter only determines the patient's body temperature, but is not a treatment tool.

It turns out that the computer attack detection system has only diagnostic value.

There are two technologies for detecting attacks, the first of which is the technology of signature analysis, and the second is the so-called technology for detecting abnormal activity. IDSs based on the first of them do not detect all attacks, but only those that are already described in the signatures (an example of an IP data packet specific to a particular attack). In other words, they only respond to known attacks and are defenseless against new, unknown ones. Such IDSs work on the same principle as anti-virus programs: known viruses are caught, unknown viruses are not.

The appearance of a new signature is always due to the analysis of the mechanism of an already passed attack and its impact on any information system. Well, if these were actions aimed at specific information resources. And if it was, say, a new kind of Internet worm? Then the user of the security system created on the basis of signature analysis technology is not immune from the consequences of a possible computer attack. The principle of "carry - will not carry" will suit only those users for whom the loss of information is not critical, but not those whose activities are based on the use of information resources.

A reasonable question arises, but what about unknown attacks? Recent computer attacks, such as Slammer or Nimda, have escaped the attention of signature-based software and are spreading almost instantly through local networks – long before any update to the security system becomes possible. A system focused on identifying new types of attacks is a system for detecting "abnormal" behavior, which monitors network abnormalities in applications and other processes, monitors the frequency of events and detects statistical anomalies. Based on an analysis of behavior, such a system can stop both known and previously unauthorized activities. However, it also has a significant drawback – difficulties with the formulation of effective criteria for what to consider abnormal behavior and what not to consider. Combining these two technologies and thus eliminating their mutual shortcomings, you can get a means of detecting known and unknown attacks.

It is necessary not only to detect, but also to block malicious influences - the transition to proactive defense. Intrusion detection without counteraction cannot be considered an effective means of protection. By purchasing an information security system, the user aims to guarantee the protection of his information resources from malicious acts. Therefore, at the next round of the development of information technologies, faced with the constant evolution of threats and following the wishes of users, the developers of information security tools proposed replacing the IDS system to prevent computer attacks.

The core of IPS is the integration of IDS with firewalls. In addition, a prerequisite for the effective operation of IPS is to install the system "in the gap" of the network. The IPS system not only determines, but also tries to stop the attack, and can even retaliate against the attacker. The most common types of response are session interruption and firewall reconfiguration. Today, IPS is already the prevailing technology implemented in the products of almost all well-known manufacturers of information security tools. Some vendors go further and try to complement the attack prevention system with unique developments.

In this regard, the popular Virtual Patch technology developed by specialists of the Internet Security System Company working in the field of information security is interesting. Its main purpose is to block the attack exploiting the vulnerability in a particular system until the official release of the patch by the manufacturer of this system. The actual update exit speed for the Virtual Patch module is several hours from the moment the first information about the vulnerability is detected, after which updates are sent and the sensors of the attack detection system are automatically reconfigured, since the impact of the system is targeted and is not related to traditional information security tools.

As of today, internal violators are almost a greater danger than external ones, because any employee can be an attacker companies, from the average user to a senior executive. And the solution to the problem of protecting information from unauthorized exposure to internal users is impossible only by organizational, or only technical methods of protection. Only the integrated application of these methods can bring results. A comprehensive information protection system should be: continuous, planned, focused, specific, active, reliable, etc. The information protection system should be based on a system of types of self-support that can realize its functioning not only in everyday conditions, but also in critical situations.

In connection with all this interconnected complex of variables that must be taken into account in the modern information society, the fact of the interconnection of all spheres of social regulation becomes obvious. Over the past decade, the role of informatization of education in the development of the information society and their close relationship have intensified.

On the one hand, the formation of the information society significantly affects the processes of penetration of information technologies into all spheres of educational activities, on the other hand, the informatization of education, forming the information culture of members of the society, significantly contributes to its informatization.

Bezpeka's information, such as information security, which is complex, is tied to security, which is implemented in the security system. The problem is that I will clean up information, a large-scale and complex and ambitious series of important tasks. The problems of information security are permanently absorbed by the processes of penetration into the sphere of the suspension of technical problems of transmission and transmission, first of all, of calculating systems.

For today's day, three basic principles have been formulated: I'm guilty of securing information without a security:

- destruction of confidentiality information;
- loss of information and information;
- impairment of prazdatnostnost i informational and calculating systems.

Security of information is going to overtake the problem of sovereign security, if I am talking about the state, diplomatic, military, industrial, medical, financial and secret information. Great masters of this kind of information can be accessed in electronic archives, accessed in information systems and transmitted via telecomunication measures. The main authorities of information and information are confidentiality and compliance, obedience to legislation, legal, as well as organizational, technical and program methods.

THEME 4. ECONOMIC INFORMATION PROCESSING SYSTEMS AND THEIR ROLE IN THE MANAGEMENT OF ECONOMIC OBJECTS

4.1. The concept of economic systems, management systems and processing systems of economic information, their classification and structure.

Any business entity is an object of management carried out by the management body, which provides a stable and targeted behavior of these entities to achieve the goal. Based on this, in the following presentation we will use the term "economic object" for an economic management object and use the following definition for it:

Economic object is a *business entity of various nature, subject to the study of its economic activity by the governing bodies to ensure sustainable and purposeful behavior of this entity to achieve the goal.*

In Fig. 7 as an example, a diagram of an economic object is presented – an **enterprise** is an economic entity created to produce products, perform work and provide services in order to satisfy social needs and make a profit.

Management bodies are divided into *external and internal governing bodies*. External governing bodies are not included in the structure of the economic object, and internal go in. Parent and regulatory bodies in Fig. 7 are the external governing bodies of the enterprise, and the administration and management services are the internal governing bodies of the enterprise. Support services, production units and sales services are business entities of the enterprise that are subject to the study of their economic activities and management bodies

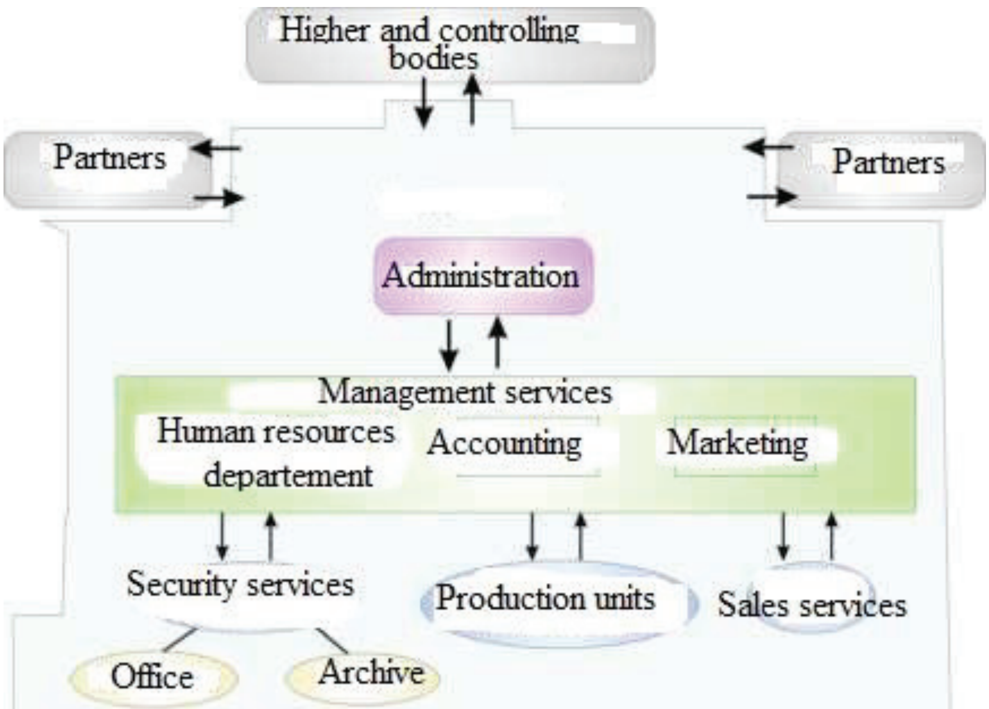


Fig. 7. The schematic structure of enterprise management

In turn, ensuring the targeted behavior of economic objects as a result of management is impossible without the availability of economic information, the promotion of which in Fig. 7 is indicated by arrows.

Economic information *– this is a set of data, the processing of which by special methods provides obtaining information of an economic nature on the basis of which the economic activity of an economic object is carried out, as well as the planning, accounting, analysis, control processes at all levels of management of an economic object.*

To analyze the economic object – the enterprise (Fig. 7) we took a systematic approach and therefore we examined the enterprise under study as an economic system. In connection with the different content and presentation of the concept of the prisoner in the term "system" (see topic 1) now we give the definition of “economic system”, revealing the essence of the direction of research of an economic object from the standpoint of systems theory and system analysis and which we will adhere to in the future:

Economic system *is a set of interconnected economic elements that form a certain integrity, the economic structure of society, the unity of relations that develop over production, distribution, exchange and consumption, an economic product.*

Unity of relationship *is a historically emerging or established, existing in the country set of principles, rules, legislatively established norms that determine the form and content of the main economic relations arising in the production, distribution, exchange and consumption of an economic product.*

Economic product *– the result of human labor, economic activity, presented in material form (material product), in spiritual, informational form (intellectual product) or in the form of work and services performed.*

Each economic system includes such ***components:***

- system structure – many elements of the system, which are subsystems in relation to the entire economic system and the relationships between them. For example, the structure of the enterprise (Fig. 7), where each element, in turn, can be considered as a system, i.e., a subsystem of the entire enterprise system.

- the functions of each subsystem of the economic system and the entire system as a whole. For example, managerial functions are decision-making in certain structural subdivisions of an enterprise and regarding the enterprise as a whole (for example, administration in Fig. 7);

- input and output of each subsystem of the system and the entire system as a whole. For example, material or information flows that enter the system or are output by it (for example, in Fig. 7, these are higher and controlling bodies, partner enterprises);

- limitations of the economic system and its individual subsystems. For example, resource constraints: material, financial, informational, etc.

It should be noted that the structure and functions of the economic system and its individual subsystems depend on the purpose for which the economic system is intended. For example, the structures of organizational and production systems, catering systems and accounting of the same enterprise are different.

Based on the above concepts and definitions, we now define management of economic systems and consider management problems in accordance with these concepts.

Control – the process of purposeful behavior of the economic system through informational impacts generated by a person (group of people) or a device – a management body.

The task of the governing body of the economic system is to develop, on the basis of incoming information about the current state of the system, control information that will allow the system to be brought into some desired state.

The main management tasks include:

• goal-setting – determining the desired state or behavior of the system;

• stabilization – keeping the system in an existing state under conditions of disturbing influences;

• program execution – transferring the system to the required state under conditions when the values of controlled quantities change according to deterministic (known and unchanged) laws and the behavior of the system can be absolutely accurately predicted;

• tracking – keeping the system on a given trajectory (ensuring the required behavior) when the laws of change of controlled quantities are unknown or change;

• optimization – keeping or transferring the system to a state with extreme values of characteristics under given conditions and limitations.

For further discussion, we highlight the subsystems of interest to us that provide the process of managing economic systems, and consider their structure and properties.

System including 3 *subsystems: a control system, a control object (business entities of the economic system) and a communication system that provides for the exchange of information flows between them, called the* ***control system*** *(CS).*

Control system *called the whole complex constituting the unity of the elements, their relationships and interactions designed to manage the economic system by decision makers and the implementation of all management tasks.*

In other words, a ***control system*** is a system in which management functions are implemented. Therefore, any ***economic system*** is a ***management system***.

For example, the structural diagram of the economic system – the enterprise depicted in Fig. 7 is an ***enterprise management system***. Administration and management services make up the management system, and support services, production units, and sales services comprise management objects (business entities of the enterprise).

The main groups of functions of CS are:

• decision-making functions – collection and transformation of information content (creation of new information);

• routine information processing functions;

• information exchange functions.

The implementation of these functions of the control system provides an ***information system***.

In the activity of an enterprise (organization), the ***information system of an enterprise*** is considered as a system for ***processing economic information***, which is software that implements the organization's business strategy where chronologically and systematically accumulate and process data related to accounting, control, planning, analysis and regulation. Based on these data, economic information on the operation of the enterprise is compiled, actual and regulatory indicators are compared, global and local areas of activity are formulated, proposals are developed for identifying the causes of deviations and correction of results, forecasting the effectiveness of the enterprise management policy is carried out, i.e., all the basic functions of the system are implemented enterprise management. That is, it is a combination of components (information, procedures, personnel, hardware and software), combined by regulated relationships to form an organization as a whole and ensure its focused activities. At the same time, the large-scale goal is to create and deploy a single corporate information system meeting the information needs of all employees, services and units of the organization.

4.2. Organization of the information base of economic information processing systems.

Any economic object is a subsystem of a larger system, that is, it is an ***open system***. For example, an enterprise, as an economic system, is a subsystem of the socio-economic system of the region, and the economic system of the region is a subsystem of the state socio-economic system. This leads to the fact that the company may set goals and non-economic nature. Therefore, the information that is necessary to manage the enterprise is a different message of economic, technical, technological, political, social, legal, demographic and other content. But nevertheless, the largest amount falls on technical and economic information, i.e. information that describes ***business processes*** *– a set of interrelated activities or activities aimed at creating a specific product or service for consumers and* *making a profit.* Since the receipt of profit ensures the achievement of other goals: social, environmental, etc.

Thus, obtaining all indicators according to the results of the enterprise's activity and their assessment are mainly based on economic information, and the main source of economic information is the ***accounting and analysis system*** – accounting information.

4.2.1. Accounting information, its content and formalization.

Accounting information is a *direct reflection of the business operations performed at the enterprise, i.e. this is "historical" information about processes that have already occurred.*

Accounting information is characterized by two main features: *qualitative* (allows you to classify it according to the field of knowledge, types and categories of management functions, etc.) and *quantitative* (is a way to evaluate or measure, on the basis of which it is possible to establish its volume and the complexity of collecting, storing and fixing as well as processing technology). Characteristic *features of accounting information* are: large volumes, reusable repetitions of acquisition cycles and transformation into certain time periods (month, quarter, year, etc.), a significant proportion of logical and mathematical operations during processing.

The advantage of accounting information is that it allows you to display the activity of an economic object through a system of numerical indicators, moreover, numerical estimates that characterize specific facts, objects, phenomena, business transactions, etc. of an economic object can be expressed in the same unit of measurement – cash equivalent and therefore amenable to formalization.

Formalization is a *stage of systematic research, at which the basic hypothesis formulated in the content description about the mechanism of the studied process of the system's functioning and its regularities takes on a strict logical and quantitative form of cause-effect relations, that is, formalized means written in the form of formulas and, therefore, provides for the use of computer technologies and computer systems for system analysis.*

The formalized description, as well as a computer system for processing accounting information for evaluating the activities of the enterprise and forecasting further activities, provides for the following **logical structure of the recorded information:**

- ***Data*** – these are symbols (letters, numbers, signs) that are recorded on a material medium (database), contain information and are intended for its processing;
- ***Attribute*** **(props)** is an elementary part of the data that is distributed on the signs and foundations. ***Details-basics*** reflect the quantitative side of objects and phenomena, details-signs – their qualitative side. Logical operations (ordering, searching, grouping, etc.) are performed on attribute attributes, and arithmetic operations are performed on essential attributes. Attributes have the following characteristics: ***name, designation in formulas, identifier, type, format, length, accuracy.*** The *name of the attribute* is natural, i.e., corresponds to its meaning. The *designation in the formulas*, as a rule, is one symbol of the Cyrillic alphabet or the Latin alphabet. It is customary to designate base attributes in capital letters, attribute attributes in small letters. Designations are used in the development of a mathematical description of the algorithm (formalized description of indicators). *Identifier* is the internal name of the attribute when creating the database (name of the database field). *Type* – attributes are quantitative (planned, actual, estimated, normative, etc.) and qualitative (grouping, reference). A *format* is a type of field in a database (text, numeric, monetary, logical, date / time, note, etc.). The ability to use one or another attribute format depends on the use of a specific database management system, i.e. from what types of fields it supports. Length - the maximum number of characters that the attribute value may occupy. *Accuracy* – this characteristic is inherent in quantitative attributes. Accuracy equals the number of digits after the decimal point;
- ***Economic indicator*** *– a structural unit that characterizes any particular control object from a quantitative and qualitative perspective, it consists*

of essential details and essential attributes sufficient to create an elementary document (enterprise turnover per month);

- A set of interconnected data of one form with all its values is an ***array of data*** (a set of data on the movement of goods in an enterprise);
- The set of data arrays, which refers to the same area of management work, is called the ***information flow***;
- The set of information about the control object represents an ***information base.***

4.2.2. Carriers of accounting information and their classification in automated accounting systems.

The rational construction of an ***Automated Accounting System*** (AAS) needs a classification of accounting information, which is divided into:

- ***incoming*** (a set of data that is necessary to solve accounting problems: primary and accumulated accounting data, data from previous reporting periods, constant reference data);

- ***outgoing*** (data obtained as a result of solving accounting problems and intended for the direct performance of accounting functions);

- ***intermediate*** (data that are the result of solving some problems and are used to solve other problems).

The combination of three types of information, which is organized in a certain way and recorded on computer media, forms an ***information database*** (**ID**) or ***database*** (**DB**).

Data in the information security is organized and stored in a certain order, so it consists of: **database**> **files**> **records**> **data aggregates**> **fields**> **symbols**.

Symbol – the lowest element, which can be expressed by a number, letter, official or special character.

Field – this is a combination of characters that creates a minimal semantic (semantic) array (product, quantity, price).

Data aggregate – this is a collection of two or more lower level elements (date of birth, address).

Record – this is a named set of fields, united by a meaningful principle, which is an object and the result of one step of data processing (employee salary, spare part code).

File – this is a set of records of a data array for objects of the same type, which contains external computer programs and accessible programs using special operations (spare parts).

Database – this is a roll-call collection of interconnected files with minimal redundancy, which is intended for simultaneous use by many users (work and personnel).

As in any automated information management and processing system, the classification of accounting information in the form of a classifier is of great importance in AAS.

Classifier *– a systematic list of named object each of which is given a unique code.*

Classification of accounting objects (materials, products, business units, employees, accounting accounts, etc.) is carried out in accordance with the rules for distributing a given set of objects into subsets (*classification groups*) in accordance with established signs of their difference or similarity.

Classifier is the standard code language of documents, financial statements and automated systems. Classification of accounting objects contributes to their systematization, processing, deeper study and the creation of uniform classifiers of homogeneous objects for different enterprises. Two systems for classifying accounting objects are known: hierarchical and faceted.

A ***hierarchical classification method*** is understood to mean a method in which a given set is sequentially divided into subordinate subsets, gradually specifying the classification object. In this case, the basis of the division is some selected feature. The totality of the resulting groupings in this case forms a hierarchical tree structure in the form of a branching graph, the nodes of which are groupings.

The choice of a sequence of features depends, first of all, on the nature of the information. When constructing a classification, the selection of a sequence of features depends on the probability of accessing a particular feature. At the same time, the highest levels of classification should correspond to the most probable requests.

***Facet classification method*:** method classification, in which a given set of classification objects is divided into independent subsets according to various classification criteria, i.e. implies a parallel division of many objects into independent classification groups. Moreover, a rigid classification structure and pre-constructed finite groups are not assumed. Classification groups are formed by a combination of values taken from the corresponding facets. The sequence of facets in the formation of the classification group is given by the facet formula. The number of *faceted formulas* is determined by possible combinations of features.

The classifier based on the faceted classification method has the following requirements:

1. The principle of non-intersection of the facet must be observed, that is, the composition of the features of one facet should not be repeated in other facets of the same class;
2. The classifier should include only such facets and features that are necessary to solve specific problems.

Accounting as a process in AAS consists of three stages:

- ***primary accounting*** – provides for the formation of primary documents (primary evidence that is used at the stage of primary accounting). In ASBU they have an electronic form and are stored in the database as auxiliary files of primary accounting. Can be displayed on the monitor screen for viewing, adjustment, transfer to other subsystems, can be printed and received on paper;

- ***current accounting*** – provides for the formation of accounting registers (secondary evidence that is used at the current accounting stage). ASBU receive an electronic form during their formation. They, as a rule, are not stored in the database, since they are formed almost instantly. Each such document is built in the form of a pre-designed layout, which provides for algorithmic communication with an array of business transactions and, accordingly, a system of synthetic analytical accounts. In paper form, such documents, in contrast to traditional (manual) accounting forms, as a rule, are not formed. The formalization of current accounting documents is arbitrary and is determined by accounting programs, the needs of users of accounting information, accounting features of an enterprise, accounting science;

- ***final (generalizing) accounting*** – provides for reporting (tertiary evidence that is used at the stage of final accounting). In ASBU they have an electronic form and are stored in the database as auxiliary files of source (reporting) documents. They can be displayed on the monitor screen for viewing, adjustment, transfer to other subsystems, can be printed and received on paper.

An important component of the system for presenting and interpreting credentials that are used to create AAS is a **workflow model**. In existing accounting programs, the following document interpretation models are used:

Addition to business transactions – it is used in programs where the most important component of the information base is an array of business transactions with certain accounting entries. The documents here are treated as input forms of primary accounting, their construction is carried out automatically or upon request after entering data on the business transaction. Primary documents are stored in a database and do not have algorithmic communications with business operations and accounting accounts.

Means of generating records of an array of business transactions – characteristic of programs in which the primary document is not only a formal basis for the formation of accounting transactions of business transactions. In such programs, there is a certain set of forms of documents that determine the structure of information input. For the data entered on the basis of the layout of a specific document, the program generates the records of the array of business transactions appropriate for the document. After that, the document loses its connection with business operations. The document is considered as an auxiliary means of maintaining records of an array of business transactions.

Supporting Information Objects – the model provides for the application of technologies for conducting primary documents, i.e., automatic display of a business transaction in the information base in the accounting system of accounts.

Full workflow model – this model does not provide for the principle of the relationship of documents and records of an array of business transactions, but provides for the maintenance of a communication system between documents of different types. The basic element is a document, along with a set of unique relationships with other documents. With such a document management system, accounting records are secondary information.

There are three options for registering credentials in AAS:

1) registration is carried out in the primary document with the subsequent transfer of this data to the machine carrier;

2) registration is carried out simultaneously in the primary document and on a machine medium (an electronic document layout is created);

3) registration of data only on a machine medium, the vast majority of primary documents at the same time arrives through electronic communication channels.

Storage media of actual accounting information can be: paper, storage on disks, storage on hard magnetic disks, storage on magnetic disks. Today, paper documents are the most common carriers of accounting information. The possible execution of operations without paper documents, when the initial information via communication channels enters the computer in electronic form and generates an electronic primary document is the most promising way to develop an automated form of accounting and the subsequent preparation of financial statements.

4.2.3. Classification of computer accounting programs.

From the point of view of software developers, the most complete is the classification of accounting programs that has developed during software competitions. This classification combines the criteria for grouping, both by purpose and by the method of implementation of accounting functions and the size of enterprises:

Home Accounting Software – with their help, PCs can be used to account for personal income and expenses, to plan the family budget (including with long-term investments), and to prepare personal tax returns.

Mina Accounting is a program that is intended for use by one or more employees in the bookkeeping of small enterprises. Such programs do not have a clear specialization in accounting areas. They implement the functions of synthetic and final analytical accounting, allow you to enter business transactions and process them (sort, search for necessary information, etc.), form a small set of primary documents and reporting forms.

Universal accounting systems (media accounting), these programs are focused on bookkeeping of small and medium-sized enterprises, which in a simplified version provide the maintenance of all areas of accounting. Such systems combine all accounting functions, including the functions of quantitative accounting, and, as a rule, are designed to work on one computer. An exception is payroll, which is performed separately. Some programs of this class are designed to work on several computers on the local network.

Local workstations designed to perform individual accounting tasks – accounting for labor and wages, fixed assets, inventory materials, etc., cover individual sections of accounting and, as a rule, are not interconnected. Workstations have a high level of specialization and therefore can effectively automate individual parts of accounting without generating consolidated reporting.

Complexes of related workstations focused on the use in accounting with more than 8 employees with a clear distribution of functions between them. The complex consists of a set of workstations, each of which implements the functions of individual accounting sections. Each such program is designed for the specifics of accounting work and is aimed at personnel with low accounting and computer skills. As a rule, automated workplaces of the complex support extensive analytical accounting, have deep specialization and are installed on separate PCs. The complex contains the means of combining data from different workstations, which are necessary to obtain consolidated reporting forms. The data are combined using a specialized central module – the General Ledger. When combining data, information can be exchanged both with the help of information storage devices and in a local network.

Management systems are complete organizational management systems with elements of accounting, planning, paperwork, as well as decision-making modules and some others. The accounting component of the program in this case is not the main one. More important is the interconnection of all the components of the system, the ability to effectively manage the enterprise, assistance in solving the main task of the business – making a profit.

Financial and analytical systems are various financial analysis programs based on accounting data, with the help of which the tasks of external and internal audit are automated.

Legal Databases these are ordered reference systems that contain thematic or chronological legislative acts on taxes, accounting, etc. These systems allow you to quickly find the documents you need, quickly track changes and additions to regulatory documents. They are indirectly related to accounting programs. Although this category of software was created primarily as an aid to lawyers, a significant part of it is aimed at users such as accountants, financiers, and economists.

The recommended sequence for choosing an accounting program is as follows:

1. Determination of the main areas of accounting.
2. Determination of the budget for the introduction of a computer accounting program.
3. Determination of the list of programs that have acceptable functionality and meet the existing technical support and cost budget.
4. Obtaining demo versions of programs for familiarization with them and evaluating their suitability.

Assessing the suitability of the obtained demo versions, determining the amount of additional costs for the acquisition of an accounting system (establishing, training personnel, improving, expanding functional capabilities).

The choice of the best option for a computer system and its implementation in the enterprise.

4.2.4. Computer forms of accounting.

For many years, the appearance of registers was considered the main sign of accounting forms. The concept of an accounting register is an integral part of determining the forms of accounting. The register is a secondary source of information intended for the formation of a predetermined list of indicators. The basis of the construction of accounting registers with paper forms of accounting is the principle of surplus, that is, all the data that may be needed is regularly generated and recorded in the registers. Registers are multi-purpose. They form the information necessary for functional services for a variety of purposes: analysis and identification of reserves, filling out reporting forms, monitoring the current and next, issuing certificates, etc.

The form of accounting is also determined by the relationships between:

- registers of synthetic and analytical accounting;
- registers and reporting;
- accounting entries in registers;
- registers of chronological and systematic records;
- accounting registers and documents;
- subtotals and input data.

The structure of each form includes the following components: recording type (simple, double), recording sequence (systematic, chronological), generalized recording (synthetic, analytical), completeness of recording.

One of the most important features of the form is the technological process of information processing, which depends on the composition of technical means, qualifications and structure of the accounting apparatus, production technology, etc.

The following factors influence the development of accounting forms:

1. The need for a wide division of labor between accounting employees;
2. The need for detailed analytical accounting (analytical accounting went through three stages of development: book, card, computerized);
3. The need to quickly obtain effective information.

Two things slow down accounting work: arithmetic counting and rewriting. In this regard, they talk about the "problem of lace" in accounting.

Openwork – the speed of processing credentials on the way from registering primary information to generating reports, or the state of the enterprise accounts in which they do not lag behind the performed business operations and give the correct picture of the activities and financial condition of the enterprise.

Household openwork – such an organization of documentation in which the preparation of primary documents occurs simultaneously with the implementation of business transactions or immediately after their implementation.

Technological lace – such an organization of the work of accountants, in which the primary documents are referred to accounting registers immediately after their preparation.

It is impossible to achieve full openwork with paper forms of accounting, since the accounting department operates with executed and verified documents, for the preparation of which it is necessary to overcome certain obstacles in time and space.

In the history of computerized forms of accounting, depending on the technical means that are used, four stages can be distinguished:

Stage 1. Use of perforating machines. For the first time, a new approach to the organization of accounting with its complex mechanization was developed in the context of the use of perforating computers in the late 50s of the twentieth century. This form of accounting is called tabular punch card. It was developed for enterprises that used punching machines, and provided for the transfer of data from each document to a machine carrier – a punch card. For each accounting section: inventory accounting, labor remuneration, finished goods, etc., punch card arrays were compiled. The principle of the continuous processing of accounting information on computing devices with the complete mechanization of all accounting work was based on the comprehensive mechanization of accounting in a tabular and punch card form. At the same time, the work was redistributed, and a significant part of the operations of the accounting process was performed by the personnel of the computer center. An essential feature of this form was that it combined the use of two types of information printed on a punch card - replaceable (one-time) and permanent (normative-reference). Documents on business transactions, drawn up, verified and accepted for processing, were recorded in a special journal for receiving documents and registering control numbers, which was intended to control the storage of these documents and to verify the completeness of entries in tabulograms. Registered documents were transferred to a punch for punching cards. Prepared punch cards were sent to computers, where they were grouped. All data recorded for a specific period, passed through an appropriate readable device for printing information and calculating control results, while the Journal of Operations was printed, which also served as a control tabulorgram. After checking the information arrays, working tabulograms were printed, the results of which were controlled by the verified results of the Operation Log. Based on the balance and turnover data, information on analytical accounts was printed. Within a month, tabulogram-registers of synthetic accounting were compiled on accounts with different intervals. At the end of the reporting period, a tabulorgram was printed – a turnover sheet for synthetic accounts. The results of the turnover sheets for the analytical accounts were compared with the data of the turnover sheets for the synthetic accounts. When compiling a tabulorgram – a turnover sheet for synthetic accounts, carriers with the original balance were automatically produced. After receiving the turnover sheet, a balance sheet was drawn up.

Stage 2. The use of third-generation computers (large and medium) and many terminal computing systems. The introduction into practice of the accounting process of electronic computers such as "Ural", "Dnepr", "Minsk" led to the creation of a table-automated form of accounting. In this form, primary data, except paper media, can be immediately recorded on machine media. It allows you to automate the collection of primary information. Machine media also stores current, reference and incoming information. Since normative and reference information is subject to repeated use, it refers to special databases. Depending on the use of technical means and storage media, there are two ways of introducing accounting information: direct and using peripheral technology. One of the most important principles of this stage in the development of computerized forms of accounting is the use of the request mode in order to obtain reports on the necessary indicators. To this end, the accountant fills out a standard document, which indicates the type of request. After that, the computer provides the necessary information.

Stage 3. Dialog-automated form. Using a PC led to the emergence of a dialog-automated form of accounting, which is characterized by complete automation of the processing and systematization of accounting information. Moreover, any data can be displayed in the account immediately after they are entered into the information database. Systematization and generalization of accounting and analytical data that are contained in the information base is carried out automatically and is displayed in the source data of accounting, control and analysis. The synthesis of data in synthetic and analytical accounting is carried out simultaneously on the basis of the same information. The dialogue-automated form provides for the automated execution of accounting tasks, both in scheduled and in request (interactive) modes. When using the query mode, the efficiency of accounting, control and analysis increases, it becomes possible to obtain the necessary reference and analytical data during the reporting period, and not only at the end of it. At the same time, the amount of information regularly issued to users is significantly reduced and limited only by the data necessary and sufficient for specific management work. If you wish, you can check the correctness of the calculations, get a transcript of each result of the indicator indicating all the incoming information and the order made in the machine calculations. At the same time, the volume of information regularly issued to users is significantly reduced and limited only by the data necessary and sufficient for specific management work. If you wish, you can check the correctness of the calculations, get a transcript of each result of the indicator indicating all the incoming information and the order made in the machine calculations. At the same time, the volume of information regularly issued to users is significantly reduced and limited only by the data necessary and sufficient for specific management work. If you wish, you can check the correctness of the calculations, get a transcript of each result of the indicator indicating all the incoming information and the order made in the machine calculations.

Stage 4. Computer communication form. The use of PCs, computing and communication networks characterizes a new stage in the use of computer technology. They allow you to effectively combine the capabilities of computers and communication lines. The database of systems is located on one computer – a LAN server. Using a single database requires communications between all computers and the server. These features are provided by the local area network. The form of accounting, which is based on the use of fourth-generation computers and electronic communications, is called the computer-communication form of accounting.

In the context of computerization, the traditional understanding of the concept of “accounting register” has changed. With paper forms of accounting, the accounting register is understood as a means intended for fixing, accumulation, systematization, generalization and display of accounting information. In the context of accounting computerization, the stage of displaying accounting information, i.e., the provision of systematized credentials in a user-friendly form, is an independent process that is not associated with the stages of accumulation, generalization and systematization, as this is carried out automatically. Primary accounting information is accumulated in the database of a computer system, and is summarized and systematized on accounts, which are represented by individual computer memory cells and are ideal, In computerized accounting, any tangible medium of accounting data from a theoretical point of view can be considered as a register, which means that any electronic media with which information is accumulated can be considered accounting registers. Registers with computerized accounting are transformed from means of generalization and grouping of information into the original forms of analytical focus.

When using computers, the register embodies the unity of three parts:

1. A computer database with a specific structure, which is designed to accumulate and store accounting information on technical media;
2. Accounting accounts, which are represented by cells in the computer's RAM and serve to systematize and summarize information;
3. Video games and typograms that are designed to display grouped and systematized accounting information.

The term "form of accounting" provides only information processing technology. The concept of "**accounting system**" is broader – it covers all the accounting objects that are displayed in it, including the display in primary documents, accounting registers and reporting. It can clearly distinguish the concept of accounting form, which corresponds to the level of accounting registers.

In conditions of computerization of the concept of the form of accounting, it passes into the **concept of an accounting system** that has such **characteristic features:**

Computerization covers all stages of accounting information processing (primary, current, final), without exception;

Efficiency (openwork) is provided in two aspects: economic and technological;

Obtaining reporting indicators occurs in the "man - computer" mode;

Distributed credential processing.

Possibility without paper accumulation and transfer of primary credentials;

In addition, the accounting system is a complex three-level system, which provides not only the technology for processing incoming information into outgoing information, but also the definition of accounting methods and the organization of work of accounting employees.

TOPIC 5. SOFTWARE ECONOMIC INFORMATION PROCESSING SYSTEMS

5.1. Information technology in the banking sector.

Currently, the development of the banking business without the use of information technology is impossible. The use of these technologies is one of the key factors in the efficiency and competitiveness of a modern bank. Therefore, financial institutions are actively investing in the automation of business processes.

Information banking technology is the process of transforming banking information based on the methods of collecting, registering, transmitting, storing and processing data in order to ensure the preparation, adoption and implementation of management decisions using personal and computer equipment. In the financial and credit system, information banking technologies contribute to the timely and high-quality performance of banking functions, as well as significantly increase the level of management of both the banking system as a whole and each bank and are a practical implementation of information banking systems.

The main banking services provided using the telecommunication environment includes:

- money transfers using international money transfer systems (Western Union, Money Gram International, etc.). Through the use of modern equipment, the latest computer technology, you can send money to anywhere in the world as soon as possible;

- non-cash payments using plastic cards, carried out through ATMs. A modern bank plastic card is not only a means of payment, but also a tool that links together various information applications and various payment environments: mobile networks, the Internet, electronic money, etc. Bank cards can be used to pay scholarships, salaries, pensions, obtaining loans, as well as with their help you can make payments on an international scale;

- an obligatory element of the electronic payment system is an ATM, which issues cash from various accounts, accepts deposits into accounts, transfers money;

- remote banking services for clients using the Internet and a mobile phone (Internet banking, mobile banking).

Currently, the following areas of development should be distinguished in the banking technology market: the Client-Bank system, Internet banking and mobile banking.

Using the Client-Bank system, bank customers can perform various operations from home or from the office: managing an account, receiving information about the status of accounts and other banking information, making payments and paying for services from settlement and other accounts and plastic cards, as well as conducting other operations.

Mobile banking – receiving banking services directly using a mobile phone or laptop using wireless access technology. This technology allows you to transfer information from Internet sites to mobile phones with Internet access. This system provides even greater freedom of access. Among the consumers of banking services using a mobile phone the first place is occupied by the Scandinavian countries, and, according to experts, in the near future more than 40% of customers will switch to mobile servicing of their accounts.

The most promising area of banking information technology development is Internet banking, the system of which consists of the following elements:

- a client using a standard web browser, with which the network is navigated, services are selected, forms are filled in, the database is operated, data operations are performed using downloadable software, client registration and account management;

- a web server providing a list of services;

- an application server located in the bank, which processes the data of customer requests and provides interaction with the automated banking system;

- a database server containing data on the status of accounts and payments of customers, reference and other information.

Thus, the successful development of the banking sector directly depends on the quality of the information technologies used. Thanks to the use of modern information technologies and telecommunication systems, banks have the opportunity to expand the banking services market, improve the quality and culture of customer service.

According to the quality of the online services provided, the following banks have the highest rating according to consumer estimates:

Security First Network Bank; Wells Fargo Bank; Citibank Salem Five Cents Saving Bank; Bank of America. The most widely used Internet banking services are presented in the countries of Northern Europe - Finland, Norway, Sweden. To date, Skandia Banken (a division of the Skandia insurance group) has been recognized as the undisputed leader in Internet banking in Sweden. Which of course speaks of their development and compliance with modern technologies.

Domestic banks have also sufficiently mastered this technology and can provide such services (for example, Privatbank, VTB Bank, Table 1):

Table 1. Types of virtual banking services

Section	Operations
1. My office	View detailed information about cards, accounts, deposits; quick operations; performing service operations on products: changing the name of the product, setting a limit on the card, blocking the card and receiving a statement on the card / account.
2. Maps	The entire list of your open cards is presented. And you can view the details and also perform any action on this card (set limits, block, make an extract).
3. Current accounts	You will find a list of open and closed current accounts. View details and manage open accounts: change the name, form and receive statements.
4. Deposits	A complete list of your open deposits is offered. View details, change names, form an extract. If you wish, you can configure the function of auto-prolongation of the deposit, change the account for payment, arrange early termination, replenish the deposit. You can also view

	closed deposits.
5. Translations	The user can create his own template for regular translations and save it for the future.
6. Payments and loans	Everything is extremely simple, you can pay bills, as well as loan amounts.
7. History	It is worth noting, since this section guarantees the legitimacy of payments, and can also inform the user about fraud frauds.

So, such services continue to conquer more and more new countries, new systems every year. The number of those who use and want to use these services is constantly growing, and that is why banks should develop and improve this service. Progress in the use of the Internet by banks is inevitable, having adapted, the bank will have sufficient additional income, otherwise it will incur enormous losses.

5.2. Information systems and technologies in investment analysis.

Investments are the key to economic development and the growth of national income. Therefore, any state and its regions is faced with the task of attracting the necessary amount of investment resources to the economy.

The opportunity to attract investment is available to regions with sufficient investment potential, which in turn depends on how well the investment attractiveness of enterprises and investment projects located in the region is evaluated. This laborious process requires the involvement of experts with specialized knowledge in various areas of the economy. Such personnel may be provided by consulting firms, but the cost of these services may not be available to many enterprises. Therefore, it becomes relevant to self-study modern methods of investment design and the use of special computer programs.

Investment analysis programs are intended only for financial calculations, which is only a small part of the work on preparing a business plan. However, it is this part that is most in need of automation and it is very difficult to execute it without the use of certain programs. In addition, the scope of some systems is not limited to the formation of a financial plan - they are comprehensive financial analysis programs.

The principle of operation of all the programs listed below is approximately the same: entering a set of parameters characterizing the project; calculation and receipt as a result of a financial report, the data of which can be examined using the analytical tools present in the program. However, there are clear differences between these programs, which are reflected in the Table. 2:

Table 2. Comparative characteristics of investment analysis software

Program	Benefits	Disadvantages
Comfar III Expert (UNIDO)	High quality techniques. UN logo. Qualitative summary of the project. Original navigation on source data.	Bad technical implementation. The absence of any binding to Ukrainian law.
Project expert (Expert Systems)	Wide functional range. Nice interface, multilingualism.	Lack of turnkey solutions. Too much attention to detail.
Investor (INEC)	Attachment to Russian law. Sophisticated technique.	Unsuitable for working with foreign investors. Inconvenient interface. Bad text conclusion.
Analyst (INEC)	Detailed methodology. Good financial conclusion.	Lack of printability. Use only the Russian language. The ill-conceived interface.
Alt-Invest 3.0 (Alto)	Based on MS Excel. Ease of reporting.	Inconvenient work with source data. Lack of project description capabilities.

Summing up the results of the comparative analysis, we can give a number of conclusions regarding the data of computer programs.

Comfar III Expert is recommended for studying financial analysis and preparing projects submitted to foreign investors. However, the taxation process has not been worked out in the program, which can negate the analytical capabilities of the program and the design results.

Project Expert is a program for professionals with high demands on the complexity of financial models. It provides great opportunities, but also makes high demands on the user.

The Investor program was created on the basis of Russian accounting and analysis standards and allows the average Russian accountant to prepare a business project without overloading with unnecessary information. For complex projects, the program is not functionally competent.

The Analyst program can be recommended for express analysis of projects, selection of enterprises for a detailed assessment and investment. In this program is ideal in terms of price / quality.

Alt-Invest is the most suitable program for those who are going to create their own methodology and reporting forms, organize the documentation preparation cycle according to their own standards. But the program is not provided with ready-made solutions.

Thus, a wide range of programs providing decision-making in the field of investment is presented on the information technology market. When choosing software, it is necessary to take into account all the advantages and disadvantages of a computer program, as well as the individual abilities and professional skills of the employee. In addition, it is necessary to note the option of using several computer programs for investment analysis, for mutual elimination of shortcomings and conducting a comprehensive analysis, taking into account all the possibilities.

5.3. Means of automating the process of financial analysis in enterprises.

Financial analysis plays an important role in the successful functioning of any enterprise. Such an analysis allows you to competently manage the enterprise, prevent bankruptcy and monitor the effectiveness of activities. In the modern world, where information and computer technologies are actively developing, more and more enterprises are resorting to automation of management processes. Specialized programs provide high analysis efficiency, convenience and analytics time saving.

The rapid development of computer and information technologies has made significant changes in decision-making related to financial issues.

The financial management system includes:

- adoption of management decisions in such a way that this leads to an increase in the financial results of the enterprise;

- the maximum removal of the credit and financial sphere from government intervention, which allowed the formation of multi sector corporations serving the financial industry;

- A startling increase in the use of computers for financial management purposes;

- increased demand for financial transactions;

- the level of inflation and the consequences of its impact on the financial sector.

In order to make a decision related to finance, organizations create an internal information network (local information systems), then, if necessary, connect to corporate, global networks. For example, such as the Corporate Network of Alfa Bank PJSC, and the SWIFT telecommunication system will be the global network.

Now you can find many software developments for the convenience of financial calculations, however, the selection of a suitable software solution is complicated and confusing. Consider some of the most popular products on the domestic market.

Audit Expert is an analytical system for diagnosing, evaluating and monitoring the financial condition of an enterprise. A distinctive feature of this program is the ability to re-evaluate all financial indicators, that is, the ability to set corrective indicators for all analyzed data separately. Audit Expert is a very flexible program, as it allows you to implement your own methods for solving any problems of analysis, diagnosis and monitoring of financial condition. The program also makes it possible to automatically receive a number of expert opinions on the financial condition, prepare reports with the necessary graphs and charts. We cannot but mention the possibility of importing data from other systems, as well as exporting results and graphs to MS Word.

Alt-Finance software product is designed to perform a comprehensive assessment of the enterprise, identify the main trends in its development, calculate basic standards for planning and forecasting, assess the creditworthiness of the enterprise. This program, like the previous one, is characterized by flexibility, namely the ability to make changes, taking into account your requirements or specific conditions. A distinctive feature of the Alt-Finance product is the ability to perform financial analysis, using not only new, but also old forms of financial reporting, as well as quick transfer from the old format to the new one.

“Olympus: FinExpert” is designed to analyze the financial condition of the enterprise and allows, based on external financial statements, not only to conduct a full analysis of the current state of the enterprise, but also to obtain, using expert and mathematical methods, an analytical assessment of its future development options. Like the previous program, FinExpert allows you to work with the old and new form of financial statements, as well as convert them from each other. It should be noted that this program works in the MS Excel environment and allows you to import data prepared in other financial systems.

Exploring the three products described above, a comparative table was compiled (Table 3).

Table 3. Comparative characteristics of software products

Criterion	Audit expert	Alt Finance	Olympus: Fin Expert
The calculation of standard financial indicators of liquidity, financial stability, profitability and business activity of the enterprise.	+	+	+
The ability to predict.	+	+	+
Ability to create your own analysis techniques.	+	+ (since version 2.0)	+
Full data translation into English.	-	+	-
Ability to create analytical reports.	+	-	+
Ability to import data	+	+	+
Validation of data entry	+	-	-

Thus, the reviewed programs have a wide range of possibilities for financial analysis and are convenient to use. However, so far there is no such program that combines all the advantages of automated systems. Nevertheless, depending on the user, his goals and type of activity, everyone will be able to find a suitable software product in the domestic market.

5.4. Information Technology Pension Fund.

The Pension Fund in the system of extra budgetary funds has a special position. In fact, it is an independent financial and credit institution, however, it can use its funds only in strictly specified areas, through the procedures regulated by the country. The country determines the size of pensions, the technology of its calculation and payments.

In other words, the pension fund accumulates funds, places them in a certain way and makes the payment guaranteed by the country.

The Pension Fund as an element of the budget system performs a distribution function and accordingly has the following distribution levels: regional, local.

The main objective of the pension fund is to manage the budget of the pension fund, which means revenue management and cost management.

Budget revenues consist of the receipt of the unified social tax, contributions for compulsory pension insurance, means of placement of temporarily free funds, as well as the amount of insurance contributions to the funded part of the retirement pension, the amount of sanctions, as well as voluntary contributions from legal entities and individuals. Control over the receipt of funds is carried out by tax authorities of the appropriate level. Collecting arrears and penalties on insurance premiums, the pension fund authorities carry out independently in a judicial proceeding.

Full accounting of pension income and expenses of the pension fund as a whole is possible only if there is a single automated information system. At the moment, there is an active development of its functional elements.

Systems have been developed for local government pension agencies that solve the following problems:

1. Collection of personal data, that is, the register of insured persons.

2. Entering and processing information about the experience and income of the insured person.

3. Obtaining extracts from the personal account of the registered person.

4. Data exchange with a database of a regional level.

5. Maintenance of classifiers, divisions and employees of the pension fund, as well as registers of the insured in the pension fund of the region by the employer and the generation of statistical reports.

If information about the employee was submitted for the first time, then an individual number of the registered one is assigned to him and a certificate is issued.

Information about the registered employee is entered into the center of the pension fund, where data on all registered employees is stored.

Further, the employer regularly informs the territorial branch of the pension fund about the experience and earnings of registered persons, independently carries out insurance contributions for each employee and transfers this information to the pension fund. This information is entered into the automated information system of the pension fund and then transferred to the general information system of the pension fund.

If necessary, the program allows you to monitor the correctness of the payment of insurance premiums by each employer.

Also, the automated information system of the pension fund allows you to conduct a statistical analysis of the work of the pension fund and generate the necessary reports in automatic mode.

Pension fund expenses are accounted for using the automated system “Assignment and Payment of Pensions and Benefits”.

The program is written in the Clipper environment and is designed for district-level sectors in a decentralized mode.

The program performs the following functions:

1. Maintaining pension case cards:

- the appointment of pensions and benefits;
- correction and recalculation of pensions;
- calculation of seniority, average earnings;
- calculation of increases and allowances;
- issuance of protocols;
- issuance of certificates of seniority, earnings, dependents.

2. Maintaining a file of recipients of alimony and benefits;

3. Registration of executive documents on institutions and surcharges and monthly calculation of the paid pension amount, taking into account these amounts;

4. Issuance of the necessary documents for the payment of pensions: statements, one-time instructions to post offices, lists of pension recipients, Sberbank branches and orders for one-time postal orders;

5. Formation of payment orders for settlements with Sberbank and post offices;

6. Mass recalculation of copies when indexing it, maintaining district coefficients, etc.;

7. The issuance of primary information on the appointment and payment of pensions;

8. Formation of forms of state statistical reporting;

9. Other supporting functions.

The software is designed to work in a Windows environment and provides simultaneous execution of several applications.

5.5. Modern software products management.

The creation of a modern enterprise is a complex and lengthy process, which can conditionally be divided into two interrelated stages. The first is the formation of a production and management structure that generates powerful flows of economic information (see topic 4). The second is the formation of a structure that manages these flows, that is, the information system of an enterprise whose main task is to develop managerial decisions. Thus, managerial activity appears in modern conditions as one of the most important factors of the functioning and development of the enterprise and represents a valuable resource of the enterprise, along with financial, material, human and other resources.

Currently, the main classes of information systems used in the practice of domestic and foreign enterprises are: systems for planning the needs of the enterprise in MRP materials and all resources (materials, capacities and finances) MRPII; ERP accounting, planning and control systems; CRM customer relationship management systems; software security for submission to the controlling authorities and the exchange of legally significant first documents between counterparties in the electronic view "MEDOC"; business process control systems, such as process control ELMA BPM. Let us consider the system data in more detail.

MRP systems (Material Requirements Planning) are software tools that automate the entire process of planning the needs of raw materials for production. The main achievement of MRP systems is to minimize the costs associated with stocks. The MRP system focuses on the use of information about suppliers, customers and production processes to control the flow of materials and components. Lots of raw materials and components are planned for entry into enterprises in accordance with the time when they will be required for the manufacture of prefabricated parts and assemblies. In turn, parts and assemblies are manufactured and delivered to the final assembly at the required time. Finished products are manufactured and delivered to customers in accordance with agreed obligations. The use of MRP systems is justified for enterprises with a relatively long production cycle of complex products and structures, consisting of numerous units and components. The main idea of MRP systems is that any accounting unit of materials or components necessary for the production of such complex products and structures should be available at the right time and in the right quantity to implement the main production plan for the production of finished products.

Initial information for the MRP system is provided by the production schedule generated by the MPS system (Master Production Schedule). Based on the MPS, the quantitative composition of the final products for each planning time period is determined. Based on this data, the MRP system performs the following basic operations:

- spare parts not included in the MPS are added to the composition of the final products;
- for MPS and spare parts, the total demand for material resources is determined in accordance with the bill of materials and the composition of the product with a distribution over planning time periods;
- the general need for materials is adjusted taking into account the stock status for each planning time period;
- the formation of orders for replenishment of stock taking into account the necessary lead times is carried out.

MRPII system (Manufactory Resource Planning), able to plan all the production resources of the enterprise: raw materials; materials; equipment with its real performance; labor costs. The difference between the MRP and MRPII systems is that the first system primarily plans material needs for production, and the MRPII system is designed to plan all the resources of the enterprise for the implementation of the production plan.

ERP-systems (Enterprise Resource Planning) are software tools that automate the entire process of accounting, planning and control in enterprises, appeared in the early 90's.In principle, they implement the same enterprise management standard as MRPII, but on a fundamentally new technical information processing platform. Effectively using ERP-systems, enterprises apply various management technologies, which can be considered as an integrated set of the following main subsystems: financial management; project management; service management; quality management; personnel Management. Each of the listed subsystems can include functional blocks, which can also be designed as separate subsystems. For example, the material management subsystem, as a rule, includes a functionally complete block “Transport management” for scheduling and transport schemes of delivery, planning and transport management. Most of these subsystems have functionality that allows you to plan material and technical resources and capacities and transform them into appropriate cash requirements. The functionality of the production management subsystems of an ERP system, as a rule, is oriented towards various types of production activities of the enterprise, the main of which are the following: discrete production; process production; implementation of projects. The first two types suggest a description in the system of the composition of the manufactured product and production technology. The latter type is more focused on planning work and resources for the implementation of long-term projects. It should also be noted

System customer relationship management CRM (Customer Relationship Management) and its modern interpretation The BITRIX24 CRM system is a complex instrumentation, which is especially suitable for the consumption of small and great business. CRM system intended for automation interaction strategies with by customers(by customers), in particular to increase sales, optimization marketing and improving customer service by maintaining customer information and a history of relationships with them, establishing and improving business processes and subsequent analysis of the results. The main goal of implementation, as a rule, is to increase the degree of customer satisfaction and loyalty by analyzing the accumulated information about customer behavior, regulating the tariff policy, and setting up marketing tools. Thanks to the use of automated centralized data processing, it becomes possible to efficiently and with minimal involvement of employees take into account the individual needs of customers, and due to the speed of processing, early identification of risks and potential opportunities. The main sources of economic effect from implementation CRM systems: increasing the number of customers served by one sales manager; reduction of losses of clients with whom the manager or employees of other departments of the company forgot to contact in time; reduction of losses due to the inability of the client to contact the company on time; the ability to cut off "cold" customers; reduced requirements for staff qualifications; increase sales.We systematize and accumulate information – a client base, document templates, leaflets with clients - a program that allows us not to interrupt the client with the input of the manager and the second time the project is activated.This approach implies that when interacting with a client, the company employee has access to all the necessary information about the relationship with this client and a decision is made based on this information (the decision information, in turn, is also saved). Among the principal benefits of implementation CRM systems can especially distinguish the following:

- increase in revenue;
- reduction of costs for sales and customer service;
- expanding the company's ability to attract new customers and retain existing ones;
- increasing customer value for the company;
- improving the level of services without attracting additional costs.

Thus, investments in CRM systems can be considered not so much as investments in equipment and software, but as investments in long-term relationships with customers, and information technologies that allow the company to effectively build relationships with customers are priority.

Program MEDOC **(My Electronic Document)**It was scattered by the Ukrainian programmers Olesya Linnik in 2010 and was designated for submission to the control of the organisation of Ukraine (DFSU, DSSU, PFU, FSS with TVP, DKSU, the Ministry of Documentation), for which there are several documents In addition, the program is necessary modules for the increase of salaries, the greatness of great companies with a pre-programmed structure and bank robots.

The roser software programs provide access code for dermal skin access. As a matter of fact, the access code for the program is not introduced, so you'll need to see the demonstration version, you can recognize the functionality of the program, aly func- tion for export, sending and receiving e-mail and other documents.

The next step is to get the program ready for the main customers, as well as add-on modules. The structure of the programs is the following:

- Solutions MEDOC-Power – the submission of all kinds of benefits to the control of an organization, the registration of patronage invoices in the Unified Register of patrimonial invoices and the exchange of counterparties with them;
- Solutions MEDOC-Biznes – exchange the first accounting documents with counterparties;
- The module MEDOC-Excise and TTN – for robots with electronic admin system realization gulf and rewarding with bill of lading;
- The module MEDOC-Salary – a lottery card and pay for employees, employees and staff management;
- The module MEDOC-Corporation – Consolidation of soundness of enterprises with a structured structure.

One of the most effective modules in MEDOC programs for managing your business is the module "Electronic Document Management". A digital document is an analogue of a sound document, of having gotten help for special program protection, as well as you can go for the first modification and retrieval (in the United Database). For singing and verification of electronic documents, a special code and a special digital signature are required.

Shifting a digital document:

1. Deny access to the single database of digital documents, so that you can improve the optimization and automation of the processes in the regional region, filtering by date, time, etc.
2. The creation of the first original documents, the victorious special templates.
3. Organizing robots with digital documents.
4. The possibility of interchange with the base of documents in the country (extended).
5. Limited access for some of the users to specific documents.
6. Audit of digital documents for additional programs in the development of tools.

7. Using Classification and Directory when working with digital documentation.

8. The ability to quickly make changes and edits to digital documents.

9. Internal and interchange of documents.

10. Using electronic digital sign-up for document cleanup for unauthorized access, re-viewing and changing.

For security of electronic document management, special codes and digital signatures are presented, as participants in this process are especially updated by the Ukrainian authority. Moreover, the MEDOC software product has successfully completed all the examinations and expertise of the DSSZI of Ukraine, so that without any serious problems, you will have to go through the ordinary business.

In particular, before the legislation of Ukraine all electronic documents are entitled to these rights, which are their analogs, and you will not be able to take advantage of them for your own benefit.

The robot system for managing business processes ELMA BPM is based on a simple idea: on this system you can model a business process organization for the help of the most important diagrams.

Then we'll get into a computer system and get healthy, so that the program allows you to control and monitor the business processes in real-life robotics business.

Such idea permits to bring the most realistic picture of the business processes into the organization to the ideal model, so I wondered.

Completed, acquired by the corners in the framework of the weekend business processes, registered in the web interface and the ELMA BPM information system at the front side of the window (completed card), all of them are presented at the table for complete presentation.

We can see the introduction of the deed – in the first place there is a card that will be replaced by the field, which can be replenished if necessary. To the whole robot for handing over the corridor, wait for the Viscount in one place – on the map; immediately vin otrimuє all necessary for the whole resource.

Whatever you want, if you don't need to have real life requirements, to get to know the ELMA BPM add-on is automatically practical: you can see the course of the business process (including the deed).

Functions of the system can be divided into 4 main groups:

- Model business process hello at the graphic editor ELMA designer, who is included in the delivery of the system.
- Automation of the process of processing the process until the next graphical model. View the starting, followed by a transition chain to the final result. Participants of the process automatically receive the endeavors in the web interface system, if the process reaches the end-user process.
- Automated monitoring and control of process execution. In case of delay of the task by subordinates, the head receives the message. It can at any time track the stage of the process and view summary reports that are automatically generated in the system.
- Fast process optimization. It is enough to make changes in the graphic model, and from the next start of process employees will receive tasks according to the new scheme.

Considering the possibilities of using the above information systems, we can conclude that they can improve the following business performance indicators:

- increase the efficiency of the entire production process as a whole;

- optimize production processes and production operations;

- reduce the costs associated with the management of production, reduce the cost of production;

- improve the quality of customer and customer service, relations with suppliers and regulatory authorities, increase the number of customers;

- to optimize the management of working capital due to a significant reduction in stocks.

THEME 6. ECONOMIC AND MATHEMATICAL MODELING AS ECONOMIC MANAGEMENT DECISION MAKING METHOD

6.1. Features of economic and mathematical computer modeling.

Making optimal decisions on managing complex economic systems has always been and remains the most important aspect of human activity (see section 3.6). There are various approaches to making optimal decisions: based on analytical abilities, common sense, intuition and the experience of the leader; collective methods – the method of “brainstorming”, “brainstorming”, etc. However, the practice of managing complex economic systems in all areas and at all levels requires the wide and effective use of quantitative mathematical methods. *The mathematical theory of making optimal (rational, focused) decisions is called the **theory of operations research**.*

In connection with this approach to the study of operations, we will adhere to the following definition:

Operations research *is a section of applied mathematics that deals with the development and application of methods finding of decisions management systems based mathematical modeling, statistical modeling and various heuristic approaches in various fields of human activity.*

In the above definition, the system is needed for a detailed preliminary analysis of the real phenomenon. The nature of the systems that appear in the above definition under the name of "organizational" can be very different, and their general mathematical models are used not only in solving production and economic problems, but also in biology, sociological research and other practical areas. Mathematics conducts a quantitative and qualitative analysis of the system, helps to predict how the system will behave under various conditions, and makes recommendations for making the best decision.

Modern methods for finding optimal solutions are focused on the use of computer tools. This makes it possible to carry out economic-mathematical modeling not only using the economic-mathematical model, which is discussed in detail in Section 3.6, but also using the simulation model.

Economically-mathematical modeling consists in the use of methods and means of mathematical modeling for the study of economic objects and phenomena.

Economically-mathematical model is a mathematical description of the studied economic object or phenomenon (the process)

Simulation model represents a single program or set of programs, allowing using a sequence of calculations and graphical display of their results, reproduce (imitate) the processes of functioning of the object, subject to the impact on the object of various, usually, random factors.

Simulation as a form of computer modeling of the economy, has been used since the early 1960s.

The essence of simulation is to display the simulated object and the dynamics of its functioning as accurately as possible, fuller, more clearly. If possible, it is necessary to deform the structure of the object as little as possible, that is, it is desirable that all parts of the object in the model have a real display, and the flows of information about them represent real flows of orders, resources, people, ideas, etc. This wish is fulfilled in to the greatest extent, if the basis for constructing a simulation model is a system analysis of the simulated object and a system synthesis of its model.

The procedure for constructing simulation models using a systematic approach involves the following steps:

1. A meaningful description of the modeling object in the form of a system, statement of the problem and formulation of goals.

2. Formalization of the problem, the construction of the structure of the model, the definition of target functions and criteria for achieving goals.

3. The choice of modeling methods for specific elements of the simulated system.

4. Model building, development of a modeling algorithm and testing on a test example, necessary model adjustment.

5. System simulation, including simulation implementation planning, simulation of input and control signals, interference, trial and unauthorized influences on certain attributes of the system, calculation of various statistical characteristics.

6. Analysis of simulation results, selection of the most effective structure and behavior strategy of the simulated system taking into account the most probable input or disturbing influences.

7. Preparation of a report on simulation modeling in terms of a given object or process and planning the implementation of simulation results in practice.

As an example of one of the first applications of simulation (in conjunction with an advanced cybernetic approach) to analyze economic phenomena, consider the dynamic model, proposed by MIT professor Jay Forrester. According to this model, an industrial enterprise can be represented by a block diagram with a number of levels (reservoirs), connected by six threads. Five of them are these are controlled threads: materials, orders, cash, equipment and labor.

Sixth stream – informational, connecting five other threads and performing a feedback function.

Flow Chart, enterprise modeling activities, complemented by a system of fairly simple algebraic equations which allow us to quantify the dynamics, i.e. changes, occurring during the course of these flows. A similar mathematical model of the enterprise allows you to evaluate, how the system will respond to the input of certain data (disturbing influences) with different parameters of delays and amplifications.

Methodology for constructing and analyzing a dynamic enterprise model, according to Forrester, includes the following six steps:

1. Specific production defined-business matter, which is subject to analysis by dynamic analysis.

2. Are formulated (in verbal expression) main relationships or causal-investigative dependencies, characterizing the structure of the studied system, and are presented in the form of a graphic flow diagram.

3. Building a mathematical model, moreover, each part of this model is created on the basis of a graphic diagram, expressing the contents of the previous stage.

4. The behavior of the simulated system or its changes over time is projected..

5. A simulation of the dynamics of the system on a computer. Calculation results, received during the program run, compared with existing data on similar real processes.

6. The model is adjusted by incorporating revised parameters or measures into it, followed by computer modeling to determine their impact on the final results.

In this way, dynamic modeling technique, proposed by Forrester, allows you to simulate the activities of the enterprise on a computer using a mathematical model, consisting of many successively solvable equations. The presence of feedback loops in the simulation model allows you to track periodic fluctuations in the level of stocks of materials, production volumes and labor force at random changes in demand for enterprise products.

Basic concepts of operations research

We give the definitions of basic concepts, used in the description of models and research methods of operations.

Operation – any controlled action (event), goal-oriented. The result of the operation depends on the method of its organization and conduct, otherwise - from selecting some modifiable parameters.

Any definite choice of variable parameters provided for achieving a goal is called ***decision***. Solutions, which for one reason or another are preferable to others, are considered ***optimal***. Therefore ***the main objective of operations research*** is a preliminary quantification of optimal solutions.

You should pay attention to that itself *decision making is beyond the scope of operations research and falls within the competence of decision makers*, which may take into account other considerations, different from mathematically sound.

General statement of the problem of operations research.

Mathematical models can be divided into ***single criteria*** and ***multi-criteria***, which are, in turn, are divided into:

- deterministic;
- stochastic (probabilistic);
- vague.

Single-criterion models use a single performance criterion, and in multi-criteria – two or more.

At ***deterministic models*** random or unknown factors are not taken into account. AT ***stochastic models*** unknown factors are considered and are random variables, for which distribution functions and various statistical characteristics are known, such as expectation, dispersion, standard deviation etc. At ***undefined models*** or, more precisely, at ***models with elements of uncertainty*** random factors are used, but there are no statistics on them, since there are no conditions for their collection.

When constructing deterministic models of operations research problems, random or uncertain factors are not taken into account. All other factors, included in the description of the operation, can be divided into the following two groups:

- ***persistent factors***, or model parameters (operation conditions), which we cannot influence, but must take into account.
- ***dependent factors***, or control variables (decision elements), which, within certain limits, we can choose at our discretion.

6.2. Examples of building economic and mathematical models.

Example 1. The economic-mathematical model of asset portfolio management. Consider the problem of investor making decisions on investing his available capital with maximum benefit for him. The set of characteristics of potential investment objects with conventional names from *A* to *F* is given in the following table.

Title	Profitability (at %)	Redemption period (year)	Reliability (in points)
A	5.5	2019	5
B	6.0	2023	4
C	8.0	2028	2
D	7.5	2020	3
E	5.5	2018	5
F	7.0	2021	4

Suppose that when deciding on the acquisition of assets, the following conditions must be met:

a) the total amount of capital to be invested is $ 100,000;

b) the share of funds invested in one object cannot exceed a quarter of the total volume;

c) more than half of all funds should be invested in long-term assets (for example, at the moment in question, these include assets with a maturity after 2022);

d) the share of assets with a reliability of less than 4 points cannot exceed one third of the total volume.

We proceed to compile an economic-mathematical model for this situation. Here OS is an investor; the goal is to maximize return on investment; the decision for the OS consists in determining the amount of investment in a particular object. All internal parameters of asset portfolio management are predetermined (i.e., are not random) and remain unchanged over time. Therefore, we are dealing with a static, deterministic mathematical model.

After identifying the most important factors, it is necessary to analyze what parameters are known (set), which parameters are unknown values; which of the parameters we can control (controlled or controlled), and which are not (unmanaged or uncontrolled parameters).

In this example, the following parameters are known: profitability, reliability, terms of repurchase. All these parameters are uncontrollable factors, but since they are specified precisely, these are fixed uncontrollable factors. Searched are the volume of investment. These parameters can be considered controllable (strategy), since the OS itself determines their value (based on real conditions). Denote the managed parameters as x_A, x_B, x_C, x_D, x_E, x_F. Then the total profit P from the placed assets, which the investor will receive, can be represented as

$$P = 0{,}055x_A + 0{,}06\,x_B + 0{,}08\,x_C + 0{,}075\,x_D + 0{,}055\,x_E + 0{,}07x_F. \quad (6.1)$$

At the next stage of modeling, we must formally describe the above limitations *a*) – *d*) on managed variables (portfolio structure).

a) Limit on total assets:

$$x_A + x_B + x_C + x_D + x_E + x_F \leq 100\,000. \quad (6.2)$$

b) Limit on the size of the share of each asset:

$$x_A \leq 25\,000,\ x_B \leq 25\,000,\ x_C \leq 25\,000,\ x_D \leq 25\,000,$$
$$x_E \leq 25000,\ x_F \leq 25\,000. \quad (6.3)$$

c) The restriction associated with the need to invest half of the funds in long-term assets:

$$x_B + x_C \leq 50\,000. \quad (6.4)$$

d) Restriction on the share of unreliable assets:

$$x_C + x_D \leq 33\,000. \quad (6.5)$$

Finally, the system of restrictions in accordance with the economic meaning of the problem should be supplemented by the conditions of non-negativity for the desired variables:

$$x_A \geq 0,\ x_B \geq 0,\ x_C \geq 0,\ x_D \geq 0, x_E \geq 0, x_F \geq 0. \quad (6.6)$$

Expressions (6.1) – (6.6) form an economic-mathematical model of investor behavior. Within the framework of this model, the task of optimizing the objective function (6.1) can be posed – the search for such variable values x_A, x_B, x_C, x_D, x_E, x_F , at which the highest value of profit is achieved (i.e., the objective function (6.1)) and at the same time restrictions on the structure of the portfolio of assets (6.2) – (6.6) are fulfilled.

We now turn to the consideration of more general models and problems.

Example 2 Economic and mathematical model production planning. Let there be some economic object (enterprise, workshop, etc.) that can produce some products of *n* kinds. In the production process, it is permissible to use *m* types of resources (raw materials). The technologies used are characterized by the norms of the costs of a unit of raw material per unit of a manufactured product. Denote by a_{ij} the amount of the *i*-th resource (*i* = 1,.., *m*), which is spent on the production of a unit of the *j*-th product (*j* = 1,..., *n*). Then all resource costs the considered enterprise (object) for the manufacture of products can be represented in the form of a rectangular matrix *A* of dimension *m* by *n*

$$A = \begin{bmatrix} a_{11} \dots a_{1j} \dots a_{1n} \\ a_{21} \dots a_{2j} \dots a_{2n} \\ a_{31} \dots a_{3j} \dots a_{3n} \\ \dots \dots \dots \dots \dots \dots \dots . \\ a_{m1} \dots a_{mj} \dots a_{mn} \end{bmatrix}$$

.

Here is the column vector

$$a^j = \begin{bmatrix} a_{1j} \\ a_{2j} \\ a_{3j} \\ \dots \dots \\ a_{mj} \end{bmatrix}$$

matrix *A* represents all the resource costs in the production of the *j*-th product, and the vector row $a_i = (a_{i1}, a_{i2}, \dots, a_{in})$ of the matrix *A* represents the costs of the *i*-th resource for all products.

If the *j*-th product is produced in quantity x_j, then within the framework of the technologies described above, we must spend $a_{1j}x_j$ the first resource, $a_{2j}x_j$– the second, and so on, $a_{mj}x_j$ – m-th. A *master production plan* for all products can be represented as an *n*-dimensional row vector $x = (x_1, x_2, \dots, x_n)$ (or column vector). Then the total costs of the *i*-th resource for the production of all products can be expressed as the sum $\sum_{j=1}^{n} a_{ij}x_j$ representing a scalar product *i*-th row vectors a_i matrix *A* and column vectors *x*. Obviously any real production system has limitations on the resources that it spends in the production process. Within the framework of the model presented, these restrictions are generated by the *m*-dimensional row vector $b = (b_1, b_2, \dots, b_m)$ (or the column vector) where b_i is the maximum amount of the *i*-th resource that can be spent in the production process. In mathematical form, these constraints are represented as a system of *m* inequalities:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m. \end{cases} \qquad (6.7)$$

Using the rules of matrix algebra, system (6.7) can be written in short form, representing the left side as the product of matrix *A* and column vector *x*, and the right side as column vector *b*:

$$A \cdot x = \begin{bmatrix} a_{11} \dots a_{1j} \dots a_{1n} \\ a_{21} \dots a_{2j} \dots a_{2n} \\ a_{31} \dots a_{3j} \dots a_{3n} \\ \dots\dots\dots\dots\dots\dots\dots \\ a_{m1} \dots a_{mj} \dots a_{mn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ x_n \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \dots \\ b_m \end{bmatrix} = b. \qquad (6.8)$$

Natural restrictions on the non-negativity of the components of the production plan should also be added to system (6.8): $x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0$ or, which is the same,

$$x \geq 0. \qquad (6.9)$$

Denoting by c_j the unit price of the *j*-th product, we obtain the expression of the total income from fulfilling the production plan specified by the vector *x*:

$$\sum_{j=1}^{n} c_j x_j = c \cdot x. \qquad (6.10)$$

Here $c = (c_1, c_2, \dots, c_n)$ is a vector, each coordinate of which is the unit cost of the corresponding manufactured product, and $c \cdot x$ is the scalar product of the vectors *c* and *x*.

Formulas (6.7) – (6.10) are nothing more than a simple economic and mathematical model that describes individual aspects of the functioning of an economic object whose behavior we want to control. In the framework of this model, generally speaking, it is possible to set various goals (tasks), but, most likely, the most "natural" goal will be to find a production plan $x \in R^n$ that gives the highest value of total income, that is, function (6.10), which is considered as an objective function and at the same time satisfies the constraint system (6.7) – (6.9). Briefly, such a task (optimization of the objective function) can be written in the following form:

$$f(x) = c \cdot x \to max, \text{где } x \in D = \{x \in R^n | A \cdot x \leq b, x \geq 0\}. \quad (6.11)$$

Despite the obvious convention of the situation under consideration and the apparent simplicity of the problem (6.11), its solution is far from trivial and in many ways became practically possible only after the development of a special mathematical apparatus. An essential advantage of the solution methods used here is their versatility, since very many economic and non-economic problems can be reduced to the model (6.11).

Example 3. Economic and mathematical model transport task. Consider the problem of organizing the transportation of a product between its production points, the number of which is m, and n consumption points. Each i-th production point (i = 1,2, ..., m) is characterized by a product stock $a_i \geq 0$, and each j-th point of consumption (j = 1, 2, ..., p) – the need for a product $b_j \geq 0$. The road network connecting the system of the points under consideration is modeled using a matrix C of dimension m by n, whose elements c_{ij} are the norms of the cost of transporting a unit of cargo from production point i to consumption point j. The cargo transportation plan in this transport network is presented in the form of an array of dimensional elements $m \times n$:

$$x = (x_{11}, \dots, x_{1n}, x_{21}, \dots, x_{2n}, \dots, x_{i1}, \dots, x_{in}, \dots, x_{m1}, \dots, x_{mn}). \quad (6.12)$$

In (6.12), the transportation plan x can be considered as a vector splitting into m groups, with n elements in each, and the i-th group corresponds to the volumes of cargo exported from the i-th production point to all possible consumption points. If there is no real carriage between points i and j, then it is assumed $x_{ij} = 0$.

Limitations on possible values $x \in R^{mn}$ have the form:

1. Restriction on the satisfaction of needs at all points of consumption:

$$\sum_{i=1}^{m} x_{ij} \geq b_j, (j = 1,2, \dots, n). \qquad (6.13)$$

2. Restrictions on the possibility of exporting stocks from all production points:

$$\sum_{j=1}^{n} x_{ij} \leq a_i, (i = 1,2, \dots, m). \qquad (6.14)$$

3. Conditions for the non-negativity of the components of the plan vector x:

$$x_{ij} \geq 0, (i = 1,2, \dots, m)(j = 1,2, \dots, n). \qquad (6.15)$$

An essential characteristic of the described model is the ratio of parameters a_i and b_j. If the total volume of production is equal to the total volume of consumption, namely,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j,$$

then the system is called balanced. When the balance condition is fulfilled, it is reasonable to impose such restrictions on the total import and export of cargo under which the whole cargo is completely exported and there are no unmet needs, i.e. conditions (6.13) and (6.14) take the form of equalities.

By analogy with the task of production planning, suppose that the cost of transportation is directly proportional to the amount of cargo carried. Then the total cost of transportation in the system will take the form:

$$f(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}. \qquad (6.16)$$

The objective function (6.16) and the restrictions described above, written in the form

$$D = \left\{ x \in R^{m \cdot n} \left| \sum_{i=1}^{m} x_{ij} = b_j, j = \overline{1,n}; \sum_{j=1}^{n} x_{ij} = a_i, i = \overline{1,m}; x_{ij} \geq 0 \right. \right\}$$

set *transport balanced economic and mathematical model.* On its basis, an optimization problem can be formulated to minimize the total cost of transportation:

$$f(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij} \to min, x \in D, \qquad (6.17)$$

which in the literature was called the *balanced transport problem in a matrix formulation.*

Example 4. Economically-mathematical model use of enterprise facilities.

The company was given a plan for the production of products by time and nomenclature: required in time T release $n_1, n_2, \ldots, n_k$ units of products $P_1, P_2, \ldots, P_k$. Products are manufactured on machine tools $S_1, S_2, \ldots, S_m$, for each of which performance is known a_{ij}(i.e. number of units P_j, which can be done on the S_i machine per unit of time) and the cost b_{ij} of manufacturing P_j products on the S_i machine per unit time.

It is necessary to draw up such a work plan for the machines (t.e. distribute output between machines), so that the production costs of all products are minimal.

We compose an economic-mathematical model of the problem. Let us denote x_{ij} the time during which the S_i machine will be engaged in manufacturing P_j products (i =1, 2,..., m; j = 1, 2,..., k).

Since the operating time of each machine is limited and does not exceed T, the following inequalities are true:

$$\begin{cases} x_{11} + x_{12} + \cdots + x_{1k} \leq T, \\ x_{21} + x_{22} + \cdots + x_{2k} \leq T, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_{m1} + x_{m2} + \cdots + x_{mk} \leq T. \end{cases} \qquad (6.18)$$

To fulfill the release plan for the nomenclature, it is necessary that the following equalities are satisfied:

$$\begin{cases} a_{11}x_{11} + a_{21}x_{21} + \cdots + a_{m1}x_{m1} = n_1, \\ a_{12}x_{12} + a_{22}x_{22} + \cdots + a_{m2}x_{m2} = n_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{1k}x_{1k} + a_{2k}x_{2k} + \cdots + a_{mk}x_{mk} = n_k. \end{cases} \qquad (6.19)$$

In addition, by the condition of the problem

$$x_{ij} \geq 0 \ (i = 1, 2, ..., m; j = 1, 2, ..., k). \qquad (6.20)$$

The cost of manufacturing all products should be minimal:

$$Z(X) = b_{11}x_{11} + b_{12}x_{12} + \cdots + b_{1k}x_{1k} + b_{21}x_{21} + \cdots + b_{2k}x_{2k} + \cdots + b_{mk}x_{mk} \to min. \qquad (6.21)$$

The economic-mathematical model of the problem on the use of capacities (equipment loading) will take the form: *find such a solution*

$$Z(X) = (x_{11}, x_{12}, \dots, x_{1k}, x_{21}, x_{22}, \dots, x_{2k}, \dots, x_{m1}, x_{m12}, \dots, x_{mk}),$$

satisfying systems (6.18) – (6.19) *and condition* (6.20), *at which the objective function* (6.21) *takes the minimum value*.

As we have already said, every task of investigating operations is an optimization one, i.e., it consists in choosing among a set of acceptable strategies those that can be qualified in one way or another as optimal for the value of the objective function that reflects the goal of controlling the system. All optimal decision making tasks can be classified in accordance with the type of objective functions, the dimension and content of the vectors *a* and *x*, which display, respectively, uncontrolled and controllable parameters. Consider the basic principles of such a classification.

6.3. Classification of research problems of economic and mathematical models.

So, we got acquainted with the general economic and mathematical model and the main task of its study, which consists in comparing the strategies with each other and choosing the best one in one sense or another. This task for the general economic-mathematical model is too complex for concrete results to be obtained. Therefore, in the general case, we can only talk about the principles of comparing strategies related to the choice of the type of performance evaluation and the corresponding concepts of optimality. Methods for finding optimal strategies, without which it is impossible to talk about the practical use of the theory of operations research, do not exist for the general model. Therefore, in the study of operations, task classes are distinguished, which are special cases of the main task for which such methods can be created.

The simplest group of operations research tasks consists of those tasks in which there are no uncontrolled factors or there are only fixed uncontrolled factors, and the efficiency criterion (usually single criterion) is defined by a numerical function.

The operations research section devoted to the study of such problems is called *mathematical programming*, and the tasks themselves are called *mathematical programming problems*. In this case, the concept of programming is used in the sense of *planning* (as opposed to computer programming).

The internal classification in the section of mathematical programming is already connected with the form of the objective function and the strategy space.

If the objective function is a numerical linear function of variables describing strategies, and the strategy space is defined by a system of linear constraints, i.e., it is a polyhedral set, then the resulting problem is called the *linear programming* problem.

If, based on the meaningful meaning of the task, its solutions must be integers, then we get the *integer programming* problem. Typically, in integer problems, the criterion and other restrictions (except for the integer condition) are assumed to be linear, therefore integer tasks can be attributed conditionally to linear programming.

If the objective function or constraints defining the strategy space are nonlinear functions, then we have a *nonlinear programming* problem. In particular, if the strategy space has convexity properties, that is, it is a convex polyhedral set, then the resulting problem is called the *convex programming* problem.

If there is a time variable in the mathematical programming problem and the efficiency criterion is not expressed explicitly as a function of strategies, but indirectly through equations describing the development of the operation process in time, then this problem relates to *dynamic programming*.

Linear programming (including integer programming), nonlinear programming (including classical extremal problems and convex programming) and dynamic programming are the main subsections of mathematical programming.

Mathematical models of the operations under study may include a set (complex) of private (local) performance criteria. Each local criterion characterizes some local goal of the decision. The case of the lack of a single criterion can be interpreted as decision making in the face of uncertainty of purpose. The choice of the optimal solution according to a set of several criteria, i.e. when the selected solution is evaluated by a set of criteria that can differ in their coefficients of relative importance (effectiveness) is the task of *multi criteria* (*vector*) optimization.

In this case, the range of feasible solutions can be divided into two disjoint parts:

- the area of agreement in which the quality of the solution can be improved simultaneously by all local criteria or without reducing the level of any of the criteria;

- an area of compromise in which an improvement in the quality of a solution according to one local criteria leads to a deterioration in the quality of a solution for others.

Obviously, the optimal solution can only belong to the field of compromises, since in the field of agreement the solution can and should be improved according to relevant criteria. The choice of the area of compromise is subjective, which narrows the scope of possible solutions. The basic schemes for choosing a compromise suggest at the outset that all local criteria are *normalized*, i.e., reduced to the same dimension or converted to a dimensionless quantity.

In the presence of uncontrolled factors of more complex species (random, when known their probability distribution and uncertain when their probability distribution is unknown) other classes of problems arise, solved by *stochastic programming* methods.

Among the tasks with uncertain uncontrolled factors, the most interesting are those in which the uncertainty is associated with the actions of other reasonable participants in the operation pursuing their goals. The operations research section that deals with the study of such problems is called *game theory*. Game problems also have their own classification, according to which several subsections of game theory are distinguished.

It should be noted that almost always when making management decisions there is a situation of ambiguity of the outcome as a result of incomplete information about the external environment. In this case, they talk about making decisions in a *risk* situation, or *risk management*. In view of the foregoing, the incompleteness of information is associated with the presence of random or uncertain uncontrolled factors. Sometimes risk is understood in a narrower sense as a combination of various circumstances and conditions that create an environment for which it is possible to determine the probability of a particular outcome. From this point of view, certainty and uncertainty are two extreme cases of completeness of the initial data, and risk is an intermediate situation.

Among the tasks of investigating operations with random uncontrolled factors, we can single out *queuing problems*. These tasks are characterized by a certain type of performance criteria and random variables that reflect their meaningful meaning.

In this tutorial, we will consider from mathematical programming only some aspects of linear programming, which forms the basis of mathematical programming.

THEME 7. METHODS OF FINDING THE OPTIMAL SOLUTION FOR THE ECONOMIC-MATHEMATICAL MODEL: LINEAR PROGRAMMING

7.1. Statement of the problem of linear programming.

Line programming – a mathematical discipline devoted to the theory and methods of finding extreme (largest and smallest) values of the objective function of a mathematical model on sets of n-dimensional vector space R^n, when the objective function and all the constraint functions are linear, and all variables are real numbers.

This mathematical model describes a lot of economic problems, which served as the basis for the creation of the mathematical apparatus of linear programming, for example, the problems that we examined in section 6.2.

Many properties of linear programming problems can also be interpreted as properties “polyhedra» in n-dimensional spaces and thus geometrically formulate and prove them.

In general, the mathematical model of the linear programming problem (further LPP) can be formulated as the problem of finding extreme values $f(x)$ objective linear numerical function *f*:

$$f(x) = \sum_{j=1}^{n} c_j x_j \to \max\ (\min), \tag{7.1}$$

Where $f(x) \in R,\ x = (x_1, x_2, \dots, x_n) \in D \subset R^n$.

The domain *D* is defined by a solvable system of equalities and inequalities, which are constraints imposed on the variables of the objective function:

$$\sum_{j=1}^{n} a_{ij} x_j (\le; =; \ge) b_{ij},\ \ \forall i = 1,2,\dots,m = \overline{1,m},\ \ x_j \in R \tag{7.2}$$

and possibly limitations

$$x_{j_1} \ge 0, x_{j_2} \ge 0, x_{j_3} \ge 0, \dots, x_{j_k} \ge 0,\ \ k \le n. \tag{7.3}$$

Conditions (7.3) are called non negativity conditions. If system (7.2) is unsolvable, then *D* is an empty set. Regarding the direction of the sign of inequality in system (7.2), we will assume that the left side is less than or equal to the right. This can be achieved by multiplying by (–1) both sides of those inequalities that have the opposite sign.

Additionally, it should be noted that the choice of the type of the sought extremum (maximum or minimum) is also relative. So, the problem of finding the maximum value of the linear objective function $f(x) = \sum_{j=1}^{n} c_j x_j$ is equivalent to the problem of finding the minimum value of a function $-f(x) = \sum_{j=1}^{n}(-c_j)x_j$.

Often the conditions of problem (7.1) - (7.3), containing restrictions only of the type of inequality, can be conveniently written in a reduced matrix form

$$f(x) = c \cdot x \to max, x \in D = \{x \in R^n | A \cdot x \leq b, x \geq 0\},$$

Where $c = (c_1, c_2, \dots, c_n)$ and $x = (x_1, x_2, \dots, x_n)$ are vectors from the space R^n, b = $(b_1, b_2, \dots, b_m)$ – vector from the space R^m, $A = (a_{ij})$ – is a real matrix of dimension m×*n*. The condition $x \geq 0$ means that the non-negativity restrictions are imposed on all coordinates of the vector *x*: $x_j \geq 0,\ \forall j = \overline{1,n}$.

A linear programming task written in the form (7.1) – (7.3), called the general task of linear programming (GTLP).

If all the constraints in the linear programming problem are equations and non-negativity conditions are imposed on all variables x_j, and the value $f(x)$ of the linear objective function f is maximized, then it is called the linear programming problem in *canonical form*, or the canonical linear programming problem (GTLP). In matrix form GTLP can be written in the following form:

$$f(x) = c \cdot x \to max, x \in D = \{x \in R^n | A \cdot x = b, x \geq 0\}. \qquad (7.4)$$

Since any optimization problem is uniquely determined by the objective function f and the domain of its definition *D*, on which the extremum is sought, we denote this problem by the pair (*D, f*).

The question may arise: are specialized methods necessary to solve the problems under consideration? Is it possible, as is customary in mathematics, to simply differentiate $f(x)$by arguments x_j, equate derivatives to zero and solve the resulting system of equations? It turns out this can not be done! Since the objective function f is linear with coefficients other than zero, therefore its derivatives with respect to all variables are constant numerical values other than zero. In this regard, the maximum (minimum) value $f(x)$ linear objective function f is achieved at the boundary of the domain of its definition *D*, i.e., where restrictions (7.2) and (7.3) begin to apply. To solve such problems, a linear programming mathematical apparatus was created, which allows one to find, if it exists, the optimal solution $x^* = (x_1^*, x_2^*, \dots, x_n^*)$ at the boundary of the region definitions *D* for which value $f(x)$ linear function f reaches a maximum or minimum: $\max f(x)\left(\min f(x)\right) = f(x^*)$.

We agree on the terminology that is used in the future and is generally accepted in the theory of linear programming.

Every vector x from the space R^n is called a TLP *plan*.

Valid plan – a plan of TLP is called that satisfies constraints (7.2) and (7.3), i.e., is contained in the domain of definition *D*. The domain *D* itself is called the *domain of admissible plans*. *An optimal plan* x^* is an acceptable plan in which the value of the objective function reaches an extreme value, i.e. a plan satisfying the condition max $f(x)$ (or min $f(x)$) = $f(x^*)$. The value $f^* = f(x^*)$ is called the *optimal value of the objective function.*

Problem solving a pair (x^*, f^*), is called, consisting of the optimal plan and the optimal value of the objective function, and the solution process consists in finding the set of all solutions of the TLP.

The vast majority of known methods for solving linear programming problems are designed for canonical problems. Therefore, the initial stage of solving any general linear programming problem is usually associated with reducing it to some equivalent canonical problem.

The general idea of moving from GTLP to CTLP is quite simple:

- *constraints in the form of inequalities are converted into equations by adding dummy non-negative variables,* x_i', $(i = 1, 2, \dots, m = \overline{1, m})$ *which simultaneously enter the objective function with a coefficient 0, i.e., do not affect its value;*

- *variables that are not subject to the condition of non-negativity are represented as the difference of two new non-negative variables:* $x_j = \overline{x}_j - \overline{\overline{x}}_j$ $\left(\overline{x}_j \geq 0, \overline{\overline{x}}_j \geq 0\right)$.

We illustrate the application of the above recommendations by an example. Let the linear programming problem (*D,f*) be given in general form with the objective function

$$f(x) = 5x_1 + 3x_2 + x_3 + 2x_4 - 2x_5 \to max,$$

and the set of feasible plans *D* defined by a system of equations and inequalities,

$$\begin{cases} 2x_1 + 4x_2 + 5x_3 = 7, \\ -3x_2 + 4x_3 - 5x_4 - 4x_5 \leq 2, \\ 3x_1 - 5x_3 + 6x_4 - 2x_5 \leq 4, \\ x_1 \geq 0, x_3 \geq 0. \end{cases}$$

Then, in accordance with the formulated rules, the equivalent canonical problem will have the form (), where D', f' :

$$f'(x') = 5x_1 + 3\overline{x}_2 - 3\overline{\overline{x}}_2 + x_3 + 2\overline{x}_4 - 2\overline{\overline{x}}_4 - 2\overline{x}_5 + 2\overline{\overline{x}}_5 + 0x_1' + 0x_2' \to max,$$

and set D' defined as:

$$\begin{cases} 2x_1 + 4\overline{x}_2 - 4\overline{\overline{x}}_2 + 5x_3 = 7, \\ -3\overline{x}_2 + 3\overline{\overline{x}}_2 + 4x_3 - 5\overline{x}_4 + 5\overline{\overline{x}}_4 - 4\overline{x}_5 + 4\overline{\overline{x}}_5 + x_1' = 2, \\ 3x_1 - 5x_3 + 6\overline{x}_4 - 6\overline{\overline{x}}_4 - 2\overline{x}_5 + 2\overline{\overline{x}}_5 + x_2' = 4, \\ x_1 \geq 0, \overline{x}_2 \geq 0, \overline{\overline{x}}_2 \geq 0, x_3 \geq 0, \overline{x}_4 \geq 0, \overline{\overline{x}}_4 \geq 0, \overline{x}_5 \geq 0, \overline{\overline{x}}_5 \geq 0, \\ x_1' \geq 0, x_2' \geq 0. \end{cases}$$

In real-life tasks, variables are often constrained by the form $\beta_j \leq x_j \leq \gamma_j$ where $\beta_j \geq 0$ and $\gamma_j \geq 0$ (they are called bilateral restrictions). Consider what to do in this case. We introduce a variable $x_j' = x_j - \beta_j$. Then there will be $x_j' \geq 0$, which gives limitations in standard form. By introducing an additional non-negative variable $x_j'' = \gamma_j - x_j$, we can write a two-sided constraint $\beta_j \leq x_j \leq \gamma_j$ as

$$x_j' + x_j'' = \gamma_j - \beta_j,\ \ x_j' \geq 0, x_j'' \geq 0. \qquad (7.5)$$

Once in the original ratios instead x_j the expression will be substituted $x_j = x_j' + \beta_j$ and constraint (7.5) is added, the problem will take on a canonical form.

It is easy to notice that the “payment” for the transition from the general form of the linear programming problem to the canonical is the growth of its dimension, which, all other things being equal, is a factor complicating the process.

7.2. The basic concepts of linear algebra and convex analysis used in the theory of mathematical programming.

We briefly recall some fundamental definitions and theorems of linear algebra and convex analysis, which are widely used in solving problems of both linear and nonlinear programming.

The fundamental concept of linear algebra is a *linear (vector) real space* over the field R of real numbers. By it is meant the set K, which is endowed with an internal law (+) – addition and an external law (·) – multiplication by a real number from the field R, having the following properties:

1. Addition on the set K is endowed with the internal law of a commutative group. $\forall x \in K$, $\forall at \in K$ and $\forall z \in K$ we have:

$x + y = y + x$;

$x + (y + z) = (x + y) + z$;

$\exists e \in K$, such that $x + e = e + x = x$ (neutral element relative to the operation),

$\exists \overline{x} \in K$ such that $\overline{x} + x = x + \overline{x} = e$ (a symmetric element with respect to the operation).

2. The external law of multiplication, such that $\forall x \in K, \forall at \in K$ and $\forall \lambda \in R$ $\forall \mu \in R$ we have:

$\lambda \cdot (x + y) = \lambda \cdot x + \lambda \cdot at$;

$(\lambda + \mu)\cdot x = \lambda\cdot x + \mu\cdot x;$

$\lambda\cdot(\mu\cdot x) = (\lambda\cdot\mu)\cdot x;$

$\varepsilon\cdot x = x$ where $\varepsilon = 1$ neutral multiplication element in the field R.

Elements from the vector space K are called vectors (or points), and its components are called the coordinates of the vector.

Particular cases of real finite-dimensional linear spaces are: Rn – vector $\vec{a}$ in which there is ordered collection of any n real number $(a_1, a_2, \dots, a_n)$: $\vec{a} = (a_1, a_2, \dots, a_n)$, which are its coordinates, and n called the dimension of space; V_3– vector $\vec{a}$, in which is a free vector in geometric space. A free vector is a set of collinear directed segments in geometric three-dimensional space, equal in magnitude and directed in one direction. By establishing an isomorphism between linear spaces V_3 and R^3 over the field R using a Cartesian rectangular coordinate system or an orthonormal basis $\vec{\imath}, \vec{\jmath}, \vec{k}$ in three-dimensional geometric space in R^3, the free vector $\vec{a}$ displayed by an ordered triple of numbers (a_1, a_2, a_3): $\vec{a} = \vec{a}_1\vec{\imath} + \vec{a}_2\vec{\jmath} + \vec{a}_3\vec{k}$.

Vector $\vec{b} = \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \dots + \beta_m\vec{a}_m$, belonging to the same vector space K is called a *linear combination* of a system of vectors $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m$ or they also say that the vector $\vec{b}$ decomposed on vectors $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m$ with coefficients $\beta_1, \beta_2, \dots, \beta_m$ or, that the vector $\vec{b}$ is generated by a system of vectors $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m$. The whole set of vectors $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m$ generated by a linear combination of a system of vectors is called its linear span. The linear span of any vector system can be generated only by all linearly independent vectors contained in the given system. The number of all linearly independent vectors in the system is called its rank.

Linear space vector system $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m$ called *linearly dependent*, if there exist such coefficients $\beta_1, \beta_2, \dots, \beta_m$, which are not simultaneously equal to zero, that their linear combination $\beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \dots + \beta_m\vec{a}_m$ is equal to the zero vector (a vector whose coordinates are all zero). Otherwise the system $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m$ called linearly independent, i.e., a linear combination of these vectors can be equal to the zero vector only at zero coefficient $\beta_1, \beta_2, \dots, \beta_m$.

The maximum possible finite number of n vectors that can form a linearly independent system in a given linear space is called the *dimension of the finite-dimensional linear space,* and any system $\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n$ of linearly independent vectors in an amount equal to the dimension is called the *basis of the vector space*. The main property of the *basis* is that any vector $\vec{a}$ linear space can be represented as a linear combination of vectors of the basis of this space and this representation is unique: $\vec{a} = \gamma_1\vec{e}_1 + \gamma_2\vec{e}_2 + \dots + \gamma_n\vec{e}_n$. The coefficients $\gamma_1, \gamma_2, \dots, \gamma_n$ of the expansion of a vector $\vec{a}$ in a basis are called its coordinates in a basis. The basis allows for any finite-dimensional real linear space of dimension n over the field Rno matter what nature it is establish an isomorphism with a real linear space R^n. And therefore, when only their properties are studied, related to the operations of addition and multiplication by a number, they can be replaced by linear spaces R^n.

Any subset of a given linear space that itself has the properties of a linear space is called its linear subspace. For instance:

- R^m there is a vector subspace of the space R^n for any $m < n$ and, in turn, R^n is a subspace of R^{n+1};
- V_1 – the linear space of collinear free vectors is a subspace V_2 – linear space of coplanar free vectors, which in turn is a subspace V_3;
- the linear span of any nonzero system of vectors in a linear space is a subspace of this space.

The set H obtained by shifting some finite-dimensional linear space $L \subset R^n$ by the vector $\vec{a} \in R^n$: $H = L + \vec{a}$, called an *affine set (space).* If the fundamental property of any linear space or subspace is that it has a zero vector, then for an affine set this is not always the case. On the coordinate plane xOy in geometric space, an example of a linear subspace V_1 is a line passing through the origin, and an affine set is any line on this plane. In oriented geometric space xyz subspace V_2 is a plane passing through the origin, and an affine set is any plane in geometric space. A characteristic property of an affine set is its belonging to any line connecting any two of its points. The dimension of the affine set coincides with the dimension of the linear subspace whose shift it is obtained.

An affine set is also the set of solutions to a joint, indefinite, inhomogeneous system of m linear algebraic equations with real coefficients and n unknowns $(x_1, x_2, \dots, x_n) \in R^n$.

The set of solutions H of such a system can be represented in the form $H = \vec{y}_1 + L\vec{y}_1 = (x_1^1, x_2^1, x_3^1, \dots, x_n^1) \in R^n L$, where L is a particular solution of an inhomogeneous system, – many solutions of the corresponding homogeneous system. The set L of solutions of a homogeneous system of linear algebraic equations constitutes a linear subspace in linear space R^n over the field R dimension $n - r$ where r is the rank of the main matrix with the dimension $m \times n$ of the system $r < n$. Any solution $\vec{y}_0 \in L$ is written as a decomposition of it into basic (fundamental) solutions: $\vec{y}_{01}, \vec{y}_{02}, \vec{y}_{0(n-r)} \in L\vec{y}_0 = \mu_1\vec{y}_{01} + \mu_2\vec{y}_{02} + \dots + \mu_r\vec{y}_{0(n-r)}$.

Therefore, the set of solutions H is an affine set in space R^n, since it is obtained as a result of the displacement of the linear space $L \subset R^n$ on vector $\vec{y}_1 \in R^n$.

If a linear space R^n is considered, then affine sets of dimension 1 belonging to this space are called *straight*, and dimensions (n –1) – hyper planes. So, R^1 and R^3 are respectively a direct and a hyper plane for R^4.. Every hyper plane divides a linear space into two half-spaces.

A set V of vectors (points) of a linear space R^n is called *convex* if it contains a segment of a line connecting any two of its points, or, in other words, from the fact that $\forall\vec{a} \in V$ and $\forall\vec{b} \in V$, follows, that $\vec{x} = (1-\lambda)\vec{a} + \lambda\vec{b} \in V$, where $0 \le \lambda \le 1$. An affine set is a convex set $\vec{x} = (1-\lambda)\vec{a} + \lambda\vec{b} \in 0 \le \lambda \le 1$.

Linear combination $\beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \dots + \beta_m\vec{a}_m$ vectors is called *convex* if and $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m \beta_1, \beta_2, \dots, \beta_m \ge 0 \beta_1 + \beta_2 + \dots + \beta_m = 1$.

A set containing all possible convex combinations of points of some set *M,* called the convex hull of this set. It can be shown that the convex hull of the set *M* is the smallest convex set containing *M.*

The convex hull of a finite set of points is called a *convex polyhedron*, and the nonempty intersection of a finite number of closed half-spaces is called a *polyhedral convex set*. Unlike a convex polyhedron, the latter can be unbounded.

A point δ of a convex set *V* is called its *corner (extreme*) point if it is not an interior point for any segment whose ends belong to the set *V*. The corner points of a convex polyhedron are its *vertices*, and it itself is the *convex hull* of its vertices.

The set *K* is called a cone with a vertex at a point x_0, if $x_0 \in K$,and from the fact that some point x_1 belongs to K ($x_1 \in K x_0$), it follows that *K* contains a ray starting at and passing through x_1, i.e.

$$\{x \in R^n | x = (1-\mu)x_0 + \mu x_1, \mu \geq 0\} \subset K$$

or

$$\{x \in R^n | x = x_0 + \mu(x_1 - x_0), \mu \geq 0\} \subset K.$$

The convex hull of a finite set of rays emanating from one point is called a *polyhedral convex cone* with a vertex at a given point.

7.3. The graphical method for solving TLP on coordinate plane $x_1 O x_2$

Consider the following example. Let the problem of maximizing the value of a linear objective function be given

$$f(x) = x_1 + 3x_2 \to \max$$

region *D* permissible plans, which is given by the following system of inequalities:

$$D = \left\{ x = (x_1, x_2) \in R^2 \middle| \begin{array}{c} x_1 + x_2 \leq 6, \\ x_1 - x_2 \leq 2, \\ x_1 \leq 3, \\ x_1 \geq 0, x_2 \geq 0. \end{array} \right\}$$

Since the number of variables in the inequalities defining the domain of admissible plans *D* of the problem is equal to two x_1 and x_2, then it can be depicted on the coordinate plane (Fig. 8) $x_1 O x_2$.

Figure 8 shows that each inequality defines a certain half-plane. The corresponding areas for each constraint are indicated by dashes. The definition of these areas is illustrated by the inequality $x_1 - x_2 \leq 2$. Putting equality in this condition: $x_1 - x_2 = 2$ we thereby define a straight line on the coordinate plane x_1Ox_2. The easiest way to build this line is by two points, for which it is most convenient to choose the points of intersection of the line with the coordinate axes. In the case under consideration, this is $(x_1 = 2; 0)$ и $(0; x_2 = -2)$. This constructed line divides the entire plane into two half-planes. In one part of it $x_1 - x_2 \leq 2$, and in the other way around $x_1 - x_2 \geq 2$.

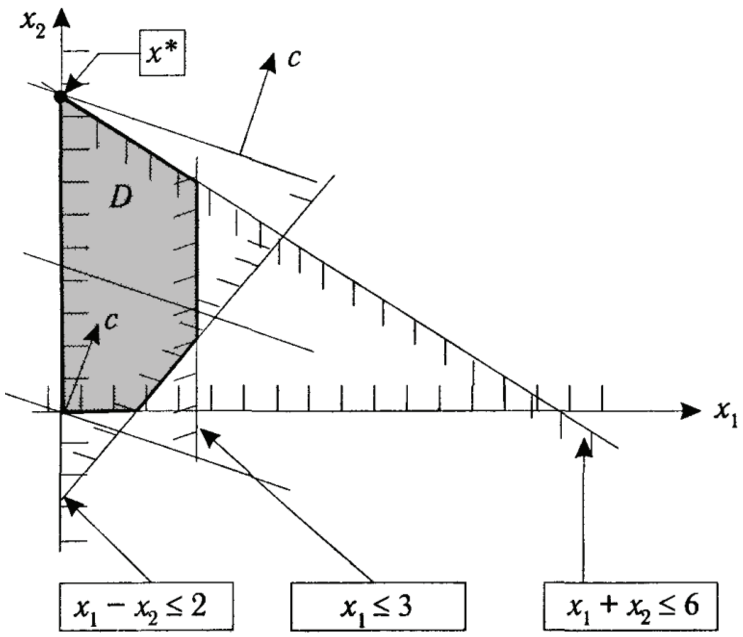


Fig. 8

To find out in which half-plane, which sign takes place, the easiest way is to look at which inequality a certain point on the plane satisfies, for example, the origin, that is, the point (0,0). In the case under consideration, this point satisfies the inequality $x_1 - x_2 \leq 2$. Therefore, all points of the half-plane containing the point(0,0) satisfy this inequality. The intersection *D* of these half-planes (i.e., the set of points that simultaneously belong to each of them) is the domain of feasible plans for the problem. Limitations $x_1 \geq 0, x_2 \geq 0$ should be noted. They cut out only the first quarter of the entire coordinate plane x_1Ox_2. Behavior in area *D* of the value of the objective function $f(x) = x_1 + 3x_2$ as part of a two-dimensional illustration, can be characterized using level lines.

Recall that *the level line of a function* is the set of points from its domain of definition at which the function takes the same fixed value. A vector oriented perpendicular to level lines indicating the direction of the most rapid increase in value $f(x)$ scalar function is called function *gradient* and is denoted by $\nabla f(x)$ (or grad *f*(x)). For a numeric function *f* many variables $x = (x_1, x_2, \dots, x_n)$ the vector $\nabla f(x)$ is an *n*-dimensional vector with coordinates: $\left(\frac{\partial f(x)}{\partial x_1}, \frac{\partial f(x)}{\partial x_2}, \frac{\partial f(x)}{\partial x_3}, \dots, \frac{\partial f(x)}{\partial x_n}\right)$.

$$\nabla f(x) = \left(\frac{\partial f(x)}{\partial x_1}, \frac{\partial f(x)}{\partial x_2}, \frac{\partial f(x)}{\partial x_3}, \dots, \frac{\partial f(x)}{\partial x_n}\right)$$

For a numerical linear function *f* two variables x_1 and x_2 the level line is a straight line on the coordinate plane x_1Ox_2, and if it is given by the equation $f(x) = c_1x_1 + c_2x_2 = const$, then the coordinates of the gradient $\nabla f(x)$ there are coefficients of a linear combination of this function: $\nabla f(x) = (c_1, c_2)$.

Thus, from a geometric point of view, the maximization problem is reduced to determining such a point of the domain *D* through which the level line passes, corresponding to the largest possible value. The latter means that in order to find the extremum point in a linear programming problem, we must first construct a level line for some arbitrary value of the objective function, for example, passing through the origin, putting $f(x) = 0$. Then it is necessary to carry out its parallel movement (so that it remains perpendicular to the vector $\nabla f(x) = (c_1, c_2)$ until we reach a point in the region of admissible plans *D* from which the displacement in the direction of the vector $\nabla f(x) = (c_1, c_2)$ it would be impossible. This representation of TLP received the name of the ***graphic*** or the ***first geometric interpretation.***

Note that the solution to the problem of finding the minimum of a linear function is carried out similarly, with the only difference being that the movement along the level lines should be in the direction opposite to the gradient of the objective function, i.e., along the vector $-\nabla f(x) = (-c_1, -c_2)$. As an example, consider the paint problem.

To paint the room you must buy 15 kg paints. This paint can be bought in banks of two types: 1,5 kg worth 10 rubles each or banks weighing 0.9 kg worth 8.5 rubles. For transportation, a box is used, which can fit 8 cans of the first type or 25 cans of the second type.

It is necessary to give a mathematical formulation of the problem of minimizing the cost of purchase. Find how many whole cans of each type you need to buy.

It is necessary to give a graphical interpretation of the solution. Compare with the solution that will turn out if a can of paint of the first type costs 17 rubles.

Decision.

Denote x_1 – the number of cans of the first type, x_2 – the number of cans of the second type. Since the amount of paint purchased should not be less 15 kg then $1{,}5x_1 + 0{,}9x_2 \geq 15$

Each can of the first type occupies 1/8 of the volume of the box, and the second 1/25 part, so the restriction on the volume of the box can be expressed as follows:

$$\frac{x_1}{8} + \frac{x_2}{25} \leq 1.$$

The purchase price is denoted by L, it is equal $L = 10x_1 + 8{,}5x_2$. We get the task (D, f) LP given by a mathematical model:

$$f(x) = L = 10x_1 + 8{,}5x_2 \rightarrow \text{min},$$

$$D = \left\{ x = (x_1, x_2) \in R^2 \;\middle|\; \begin{matrix} 1{,}5x_1 + 0{,}9x_2 \geq 15, \\ \frac{1}{8}x_1 + \frac{1}{25}x_2 \leq 1, \\ x_1 \geq 0, x_2 \geq 0. \end{matrix} \right\}$$

If banks cannot be opened, then it is necessary to add restrictions: x_1 and x_2 – natural numbers. In this case, it is a *linear integer programming* problem.

We consider the equalities

$$1{,}5x_1 + 0{,}9x_2 = 15 \text{ или } \frac{x_1}{10} + \frac{x_2}{16{,}6} = 1,$$

$$\frac{x_1}{8} + \frac{x_2}{25} = 1.$$

These are the canonical equations of straight lines. We construct them (Fig. 9). Checking the belonging of the point (0,0) of the half-plane given by each of the inequalities, we determine the region of feasible solutions. In Fig. 9 this area is hatched.

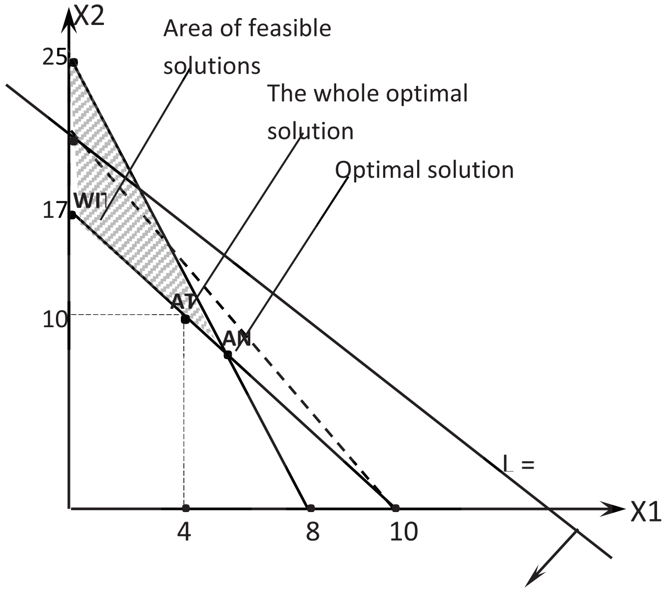


Fig. 9

We draw a line of a constant level of the objective function L = const. Take, for example, L = 170, we get the equation of the line $170 = 10x_1 + 8{,}5x_2$. We bring it to canonical form and construct $\frac{x_1}{17} + \frac{x_2}{20} = 1$.

We make sure that if $L \rightarrow \min$, then this line moves parallel to itself to the origin. The optimal point will be the exit point of this line from the range of feasible solutions.

Obviously, point A is optimal for the case when the banks can be opened (x_1 and x_2 can be non-integral), and point B is the optimal point for the whole solution.

Find the coordinates of point A, solving the equations together:

$$\begin{cases} 1{,}5x_1 + 0{,}9x_2 = 15, \\ \dfrac{1}{8}x_1 + \dfrac{1}{25}x_2 = 1. \end{cases}$$

We get $x_1^* = 5{,}8;\ x_2^* = 7{,}1;\ L^* = 118{,}35.$

Note that for the whole solution a “rounded” solution $x_1 = 6;\ x_2 = 7$ is not valid (does not satisfy the constraint $\frac{1}{8}x_1 + \frac{1}{25}x_2 \leq 1$).

The optimal whole solution, which is the last one at the exit of the direct level $f(x) = 10x_1 + 8{,}5x_2 = \text{const}$, is found from the integer feasible solutions. This is point *B* with coordinates; ; $x_1 = 4x_2 = 10L = 125$.

If the cost of a can of paint of the first type rises to 17 rubles, then the objective function will be $f(x) = L = 17x_1 + 8{,}5x_2 \to \min$.

Let's build a line of a constant level for *L* = 170: $\frac{x_1}{10} + \frac{x_2}{20} = 1$. In fig. 9 this line is dotted.

Obviously, the optimal point will be at point C : $x_1 = 0; x_2 = 16{,}6$.

The whole optimal solution will be $x_1 = 0;\ x_2 = 17$; L '= 144.5.

Answer:

1) If banks can be opened, then you need to take

– 5.8 cans of the first type;

– 7.1 cans of the second type, cost $L^* = 118{,}35$.

2) If you can’t open banks, then you need to take

– 4 banks of the first type;

– 10 cans of the second type, cost *L* = 125.

If the cost of the first type of jar becomes 17 rubles, then the optimal solution will change: $x_1 = 0;\ \ x_2 = 17$; L '= 144.5.

Using the above problems as an example, we formulate an *algorithm for graphically solving a similar type of linear programming problem (including integer):*

1. On the plane, set the coordinate system x_1Ox_2 and build on it for each inequality a system of constraints when replacing it with equality straight lines.

2. Find on the coordinate plane the region of feasible solutions to the constraint system of the problem D.

3. Construct a straight level line for a given value of the objective function *f*(x) and indicate on it the direction of its movement in the direction of the vector

grad $f(x)$ if the problem is solved to the maximum of the objective function and in the opposite direction, if to the minimum.

4. Move the level line of the objective function in the direction grad $f(x)$ (or in the opposite direction for the minimum problem) through the region of feasible solutions (RFS) so that it becomes tangent to the ODR from the secant and passes through the farthest (or closest) from the origin of the point. This point will be an extremum point, i.e., a solution to the problem.

If the level line of the objective function is parallel to one of the sides of the RFS, then the extremum is reached at all points of the corresponding side, and the linear programming problem has countless solutions.

5. Find the coordinates of the extremum point and the value of the objective function in it. If the obtained values are not integer, and in the problem you need to find an integer solution, then you need to go to the next step.

6. Select in the RFS relative to the point of extremum the region containing the optimal integer solution to the problem and limited by rounded integer values of the coordinates of the extremum point D^*.

7. Find corner points areas of D^* with integer values of the desired variables, substitute the objective function into the equation and find its value. The maximum of the obtained values of the objective function will be the solution to the problem.

In Fig. 8 and 9 shows some special cases for which the solution to the TLP is reached at the corner point of the region of admissible plans *D*, which is a bounded convex polygon. It is easy to imagine that other options are possible. They are shown in Fig. 10.

Fig. 10,(a) illustrates the situation of unboundedness of the region of admissible plans *D* of the objective function, i.e., no matter how much we move along the level lines in the direction of the vector $\nabla f(x) = (c_1, c_2)$, value $f(x)$ objective function will increase. A similar situation, for example, will turn out if the restriction $x_1 + x_2 \leq 6$ is removed in the above example (Fig. 8). The remainder will be an unbounded convex polygon.

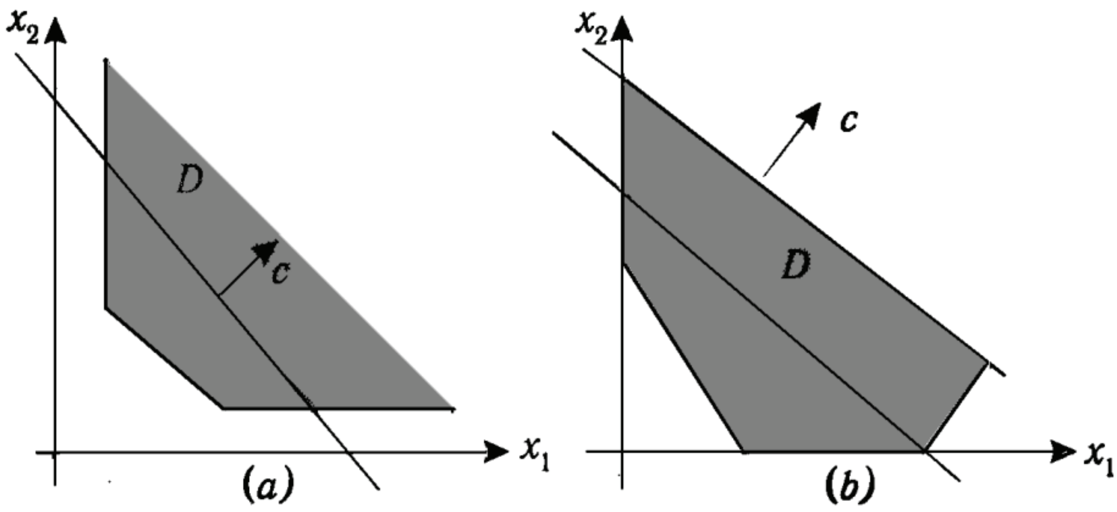


Fig. 10

In the case shown in Fig. 10, (b), the level line corresponding to the maximum value $f(x)$ touches the face of the set D, and accordingly, all points lying on this face are optimal plans.

In all the illustrations considered, the permissible TLP plans were presented in the form of some polyhedral convex set in the plane. Such a representation of them in the literature was called the first geometric interpretation of the linear programming problem.

We also note that in the same way interpretations of the TLP can be constructed for the case of the three-dimensional space R^3, where the set *D* will correspond to some bounded or unlimited polyhedron, and the behavior of the objective function will be characterized by level surfaces (planes).

Despite its obvious limitations, the graphical method of solving the TLP is often useful. In particular, it can be applied not only to problems with two variables and constraints in the form of inequalities, but also to canonical problems of the form (7.4), in which $n - r = 2$, where *n* is the number of variables, *r* is the rank of the matrix *A* of restrictions – the number of linearly independent rows or columns in the matrix.

Let be $f(x) = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ have a meaning CTLP objective function, and the domain of admissible plans *D* is defined by a system of linear algebraic equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m, \end{cases}$$

provided that $x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0$.

According to the condition, the rank *r* the main matrix $A = \begin{pmatrix} a_{11}a_{12} \dots a_{1n} \\ a_{21}a_{22} \dots a_{2n} \\ \dots\dots\dots\dots\dots \\ a_{m1}a_{m2} \dots a_{mn} \end{pmatrix}$ is $n-2$. Therefore, in this system $n-2$ basis variables, the coefficients at which form the basis minor of matrix *A*, can be expressed in terms of the linear functions of two free variables whose coefficients are not included in the basis minor. Let these free variables be x_1 and x_2 then resolving the system with respect to basis variables, we obtain $x_3 = \varphi_1(x_1, x_2)$, $x_4 = \varphi_2(x_1, x_2)$, ... $x_n = \varphi_{n-2}(x_1, x_2)$, where $\varphi_1(x_1, x_2)$ are linear non-negative functions. Substituting these values into the objective function and performing the transformations, we obtain an equivalent problem with the objective function of two variables $f'(x) = \alpha_1 x_1 + \alpha_2 x_2$, which can be solved graphically.

Before moving on to problems with an arbitrary number of variables, we formulate some theorems that reflect the fundamental properties of linear programming problems and underlie the methods for solving them. They essentially generalize to the case of problems with an arbitrary number of variables those properties that we observed in the two-dimensional case:

Theorem 7.3.1. *If the objective function f takes a maximum value at some point of the set of feasible plans D, then it takes this value at some corner point of the given set;*

Theorem 7.3.2. *If the objective function f takes a maximum value at several points of the set D, then it takes the same value at any point that is their convex combination.*

7.4. Basic solutions and geometric interpretation of CTLP.

Consider the canonical linear programming problem:

$$f(x) = c_1x_1 + c_2x_2 + \cdots + c_nx_n \to \max;$$

$$D = \left\{ x = (x_1, x_2, \dots, x_n) \in R^n \left| \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{array} \right. \right\};$$

$$x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0.$$

Denote by a_j constraint matrix columns $A = \begin{pmatrix} a_{11}a_{12} \dots a_{1n} \\ a_{21}a_{22} \dots a_{2n} \\ \dots\dots\dots\dots\dots \\ a_{m1}a_{m2} \dots a_{mn} \end{pmatrix}$ and we will consider them as vectors of space R^m: $a_j = (a_{1j}, a_{2j}, \dots, a_{mj})$. Then each valid CTLP plan – an n-dimensional vector $x = x_1, x_2, \dots, x_n$ – corresponds to a non-negative linear combination of columns a_j, $j = \overline{1, n}$, equal to column $b \in R^m$.

$$x_1a_1 + x_2a_2 + \cdots + x_na_n = b. \qquad (7.6)$$

This representation of CTLP constraints is commonly referred to as a *vector notation.* In the coordinate matrix form, equality (7.6) in the space of row vectors has the form $xA = b$ (here x is a matrix of a row of size $1 \times n$, b is a matrix of a column of size $m \times 1$). Vectors a_j , $j = \overline{1, n}$ we will call the *vectors of requirements (conditions)* of the problem (*D, f*), the vector b – the *constraint vector*, and the matrix A – *matrix of requirements vectors* (*conditions*). The set of all non-negative linear combinations of columns a_j from a geometric point of view can be represented as a polyhedral convex cone stretched over a system of vectors a_j in space R^m or as a subset of the linear span generated by a system of vectors a_j, $j = \overline{1, n}$ with non-negative coefficients.

Accordingly, the question of the existence of an admissible plan of the problem (*D, f*) is equivalent to the question of whether vector b belongs to this cone or linear span generated by a system of vectors a_j, $j = \overline{1, n}$, and the components x_j of a certain feasible plan $x \in D$ are nothing more than the *expansion coefficients of the constraint vector of problem b in the requirement vectors* a_j.

This representation of CTLP, in contrast to the graphic or the first geometric interpretation of the TLP, is called the ***second geometric interpretation***.

In what follows, without loss of generality, we can assume that the number of equations m defining the set of feasible plans D is less than or equal to the number m of variables of the problem $(m \leq n)$ and is equal to rank matrix A of the constraint system. Indeed, if this is not so, then either the system of equations of constraints $Ax = b$ (here x and b are matrix columns of size $n \times 1$ and $m \times 1$ respectively) incompatible (and, therefore, the set D is empty), or it contains redundant (linearly dependent) equations.

So the rank of the matrix A of the constraint system is equal to m; therefore, the column vector $a_1, a_2, \dots, a_n$ in the system contains t linearly independent columns of matrix A and they form a basis in space R^m, and, therefore, to represent vector b as a linear combination, only these linearly independent columns are sufficient, and not all column vectors of matrix A of the constraint system since this is presented in expansion (7.6). This means that the remaining columns will be included in this decomposition (7.6) with zero coefficients. If, moreover, the coefficients of the linear combination turn out to be non-negative, then we obtain the so-called basic admissible plan x, for which no more than m components are nonzero. We formulate the definition of the basic plan more strictly, since this is one of the fundamental concepts of the theory of linear programming.

►Let some canonical TLP (D, f) $A = (a_{ij})$ be given, is the matrix of the constraint system of the dimension problem $m \times n$, and $\theta = \{a_{j_1}, a_{j_2}, \dots, a_{j_m}\}$ is the linearly independent system of columns of the matrix A, forming basis R^m. Denote the set of column numbers included in the system θ, by $N(\theta) = \{j_1, j_2, \dots, j_m\}$. A plan x is called a *baseline plan* of the problem (D,f) if its components corresponding to the basis columns and called the basis components are greater than or equal to zero $\left(x_j \geq 0,\ j \in N(\theta)\right)$, and all other components (*non-basic*) - equal to zero$\left(x_j = 0,\ j \notin N(\theta)\right)$.

► A basic plan x is called *non-degenerate* if all its basic components are strictly positive, and *degenerate* otherwise.

In conclusion, we formulate a theorem that interprets the concept of a basic plan in terms of the first geometric interpretation of CTLP.

Theorem 7.4.1. *Each valid base plan is the corner point of the set of valid plans D.*

The converse is also true:

If x is the corner point of the set D, then it is an admissible basic plan of the problem (D, f)

At the end of the paragraph, it is necessary to note the practical significance of the established relationship between corner points and admissible basic solutions. It allows you to formalize (and thereby significantly simplify) the process of transition from one corner point to another or, what is the same, the transition from one valid basic plan to another and serves as the basis of the most famous and widely used in practice for solving the general LP simplex problem method.

The method is universal, since it allows you to solve almost any linear programming problem.

7.5. Simplex method for finding the optimal solution for CTLP.

7.5.1. The main stages of the simplex method. The simplex method is a classic method for solving CTLP, also called the *method of sequential improvement of the plan* in the literature. The optimal CTLP solution is found by sequentially targeted enumeration of the vertices of the polygon of feasible solutions or, which is the same, by moving from one feasible basic plan to another.

The idea of such a transition from one admissible basic plan to another, in which there is an "improvement" in the value of the objective function, can be demonstrated for the case when the rank of matrix A of the constraint system is *m*

= 2 using Fig. 11. If the constraint vector *b* belongs to a cone spanned by some two basis condition $\{a_{j_1}, a_{j_2}\}$ vectors, then there exists a basis plan *x* with basis components x_{j_1}, x_{j_2} such that $b = x_{j_1}a_{j_1} + x_{j_2}a_{j_2}$. Of course, there can be several such plans, depending on the choice of the system of basis vectors. In order to distinguish them by the corresponding value of the objective function $f(x) = c_{j_1}x_{j_1} + c_{j_2}x_{j_2}$, the so-called *extended vectors of conditions and constraints* are introduced. In the general case, an extended column of conditions $\overline{a}_j$ is obtained by combining the coefficient of the objective function c_j and the column a_j:

$$\overline{a}_j = (c_j, a_{1j}, a_{2j}, \dots, a_{mj}), \qquad (7.7)$$

we define the extended constraint vector as $\overline{b} = (0, b_1, b_2, \dots, b_m)$.

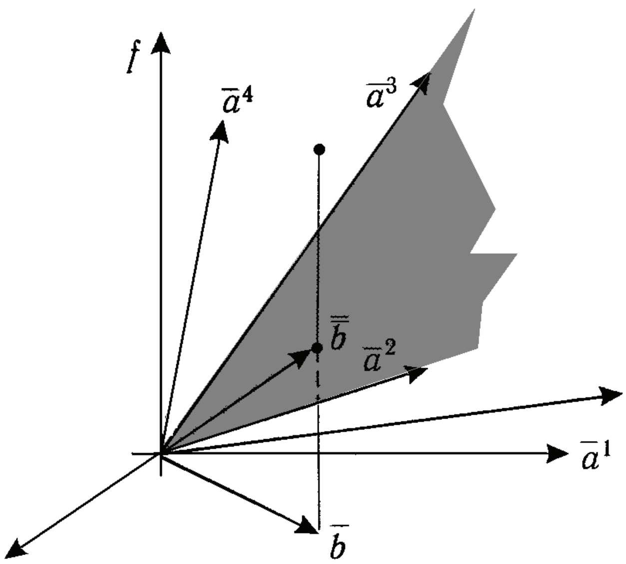


Fig. 11.

In what follows, for the convenience of numbering the elements, we assume that the added coefficient c_j of the objective function is the zero element of the *j*-th extended column of conditions, i.e. $a_{0j} = c_j$. When displaying extended vectors, the zero coordinate is plotted along the vertical axis – the *applicate axis.*

We also consider the vector

$$\overline{\overline{b}} = x_{j_1}\overline{a}_{j_1} + x_{j_2}\overline{a}_{j_2}, \dots, x_{j_m}\overline{a}_{j_m}. \qquad (7.8)$$

Geometrically, the definition of a vector $\overline{\overline{b}}$ means that it belongs to the linear span of the system of vectors $\overline{a}_{j_1}, \overline{a}_{j_2}, \dots, \overline{a}_{j_m}$, and the vector $\overline{b}$ serves as its projection. The zero coordinate $\overline{\overline{b}}_0$ of the vector $\overline{\overline{b}}$ has the form:

$$\overline{\overline{b}}_0 = x_{j_1} a_{0j_1} + x_{j_2} a_{0j_2}, \dots, x_{j_m} a_{0j_m} = f(x), \qquad (7.9)$$

i.e., it is equal to the value of the objective function for the selected basic plan.

From a geometric illustration it follows that *to solve the problem, we must among the vectors* a_j *choose one set* $\{a_{j_1}, a_{j_2}\}$ *that a straight line drawn through the end of the vector* $\overline{b}$ *parallel to the applicate axis intersect a cone stretched over the system of corresponding expanded columns* $\{\overline{a}_{j_1}, \overline{a}_{j_2}\}$ *at the "highest" point.*

In fig. 11, a cone is stretched over a system of extended columns $\{\overline{a}^2, \overline{a}^3\}$ corresponding to the current admissible basis. It is easy to see that this basis is not optimal, for example, for a basis $\{\overline{a}^3, \overline{a}^4\}$ the intersection point of the corresponding cone and line will be higher. On the contrary, you can specify a basis with the "worst" value of the objective function: $\{\overline{a}^1, \overline{a}^2\}$.

Finally, the geometric interpretation of CTLP under consideration also illustrates the general idea of the criterion that is used in the simplex method to determine the optimality (or non-optimality) of the current plan: *if there are column vectors lying above the plane passing through the vectors of the current basis, then it is not optimal and can be "improved."*

From a computational point of view, the optimality criterion in the simplex method is realized by finding special estimates for non-basis column vectors relative to the current basis.

For the convenience of the further discussion, we introduce some notation. Given that the simplex method is an iterative computational process, we denote by *q* the number of the next (current) iteration. Accordingly, the set of basis columns obtained on q-*th* iteration will be denoted as $\beta^{(q)}$:

$$\beta^{(q)} = \{a_{j_1}, a_{j_2}, \dots, a_{j_m}\}. \tag{7.10}$$

In order to make it easier to distinguish the iteration number q from the numbers of the components of the matrices and vectors, it is placed at the top and enclosed in parentheses. The numbers of the columns included in the basis are denoted by $N(\beta^{(q)})$, namely

$$N(\beta^{(q)}) = \{j_1, j_2, \dots, j_m\}. \tag{7.11}$$

In this case $N_r(\beta^{(q)}) = j_r$, the number of the column occupying the rth position in the basis. Then the current basic plan x has the form: $x = (x, x_2, \dots, x_n)$ where for $x_j = 0 j \notin N(\beta^{(q)})$.

We denote by $\overline{\Delta}(\beta^{(q)})$ a matrix composed of columns $\{a_{j_1}, a_{j_2}, \dots, a_{j_m}\}$ forming a basis, by $\Delta(\beta^{(q)})$ a matrix formed by corresponding extended columns and supplemented on the left by a column of a special form:

$$\overline{\Delta}(\beta^{(q)}) = \begin{pmatrix} -1 & c_{j_1} & c_{j_2} & \cdots & c_{j_m} \\ 0 & a_{1j_1} & a_{1j_2} & \vdots & a_{1j_m} \\ \cdots & \cdots & \cdots & & \cdots \\ 0 & a_{mj_1} & a_{mj_2} & \cdots & a_{mj_m} \end{pmatrix}, \tag{7.12}$$

and through $\Delta^{-1}(\beta^{(q)}), \overline{\Delta}^{-1}(\beta^{(q)})$ – matrices inverse to them. It also seems convenient to introduce separate notation for matrix elements $\overline{\Delta}^{-1}(\beta^{(q)})$: $\delta_i(\beta^{(q)})$ – *i*-th row of the matrix $\overline{\Delta}^{-1}(\beta^{(q)})$, $i \in \overline{0,m}$; $\delta_{ij}(\beta^{(q)})$ – matrix element $\overline{\Delta}^{-1}(\beta^{(q)})$, located in the *i*-th row of the *j*-th column, $i \in \overline{0,m}$, $j \in \overline{0,m}$.

An extended constraint vector $\overline{b}$ is represented as a linear combination of extended condition vectors with coefficients equal to the basic components of the current basic plan:

$$\overline{b} = x_{j_1}\overline{a}_{j_1} + x_{j_2}\overline{a}_{j_2}, \dots, x_{j_m}\overline{a}_{j_m}.$$

If you interpret the components of the vectors $\overline{a}_j$ and $\overline{b}$ as coordinates in an orthogonal basis, then their coordinate columns relative to an arbitrary basis $\beta^{(q)}$

supplemented by a unit vector (–1,0, ..., 0), directed opposite the applicate axis, will take the form:

$$\overline{a}_j\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{a}_j, \qquad \overline{b}\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{b}, \quad (7.13)$$

and for the expanded matrix of the problem as a whole, we can write

$$\overline{A}\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{A}.$$

The zero row of this matrix $a_{0j}\left(\beta^{(q)}\right)$ contains the coordinates of the expanded condition vectors along the applicate axis. According to the construction, the elements of this line have the following characters:

$a_{0j}\left(\beta^{(q)}\right) < 0$ – for extended condition vectors located above a plane stretched over a system of extended basis vectors;

$a_{0j}\left(\beta^{(q)}\right) > 0$ – for extended condition vectors located below a plane stretched over a system of extended basis vectors;

$a_{0j}\left(\beta^{(q)}\right) = 0$ – for extended basis vectors.

To summarize the above, we formulate an ***optimality criterion for an admissible basic plan*** in the simplex method:

►the plan is optimal if for all $j \in \overline{0,n}$; $a_{0j}\left(\beta^{(q)}\right) \geq 0$, and non-optimal otherwise, that is, if there is one $l \in \overline{0,n}$;, that $a_{0l}\left(\beta^{(q)}\right) < 0$.

Values $a_{0j}\left(\beta^{(q)}\right)$ also called column estimates of matrix *A* with respect to the current basis, or simplex differences.

If the current basis is not optimal in the simplex method algorithm, the transition to the next basis is carried out. This is done by removing one column from the basis and entering another. In order to improve the value of the objective function, a column vector with a negative estimate must be introduced into the basis. If there are several such columns, then *for input it is recommended to choose a column that has the maximum modulus estimate*. Note that this rule is relative and does not guarantee the best choice of the input column. At the same time, at this stage, it is necessary to decide which column should be derived from the basis. This must be done in such a way that the newly formed basis is valid. This requirement can be easily illustrated for the case $m = 2$. For example, in fig. 12 vectors $\{a^2, a^3\}$ form an admissible basis, and the vectors $\{a^3, a^4\}$ are invalid, since the expansion of b in a^3 and a^4contains one negative component of the plan, which contradicts the terms of the CTLP.

It can be proved that the admissibility of the basis to which the transition is carried out is ensured by the following *rule for deriving a column from the current basis:*

►*for column l, which claims to be input into the basis, and constraint vector b, we consider the relations*

$$\lambda_i = \frac{b_i(\beta^{(q)})}{a_{il}(\beta^{(q)})}, \text{где } i \in \{\overline{1,m} | a_{il}(\beta^{(q)}) > 0\}.$$

and a row r is defined such that

$$\lambda_r = \min_i \{\lambda_i\}. \qquad (7.14)$$

The resulting index r defines the column number in, derived from the basis$N(\beta^{(q)})$, *namely,* $N_r(\beta^{(q)})$.

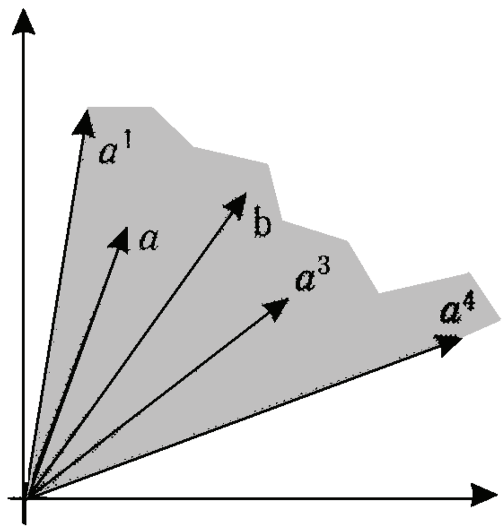


Fig. 12

Thus, if the basis at the q-th iteration included columns with numbers $N(\beta^{(q)}) = \{j_1, j_2, \dots, j_{r-1,} j_r, j_{r+1}, \dots, j_m\}$, then the basis on iteration $q + 1$ will consist of columns with numbers: $N(\beta^{(q)}) = \{j_1, j_2, \dots, j_{r-1,} j_l, j_{r+1}, \dots, j_m\}$.

We should discuss separately the case when a column $a_l\left(\beta^{(q)}\right)$ that claims to be input into basis, does not contain positive components ($a_l\left(\beta^{(q)}\right) \leq 0$).This means that the objective function in the problem is not limited to the set of admissible values, i.e., it can reach an arbitrarily large value. The latter obviously means the completion of the calculation process in view of *lack of an optimal plan.* Geometrically, the situation when $a_l\left(\beta^{(q)}\right) \leq 0$ corresponds to the fact that the ordinate axis is inside a cone stretched over a system of expanded columns $\overline{a}_j$, which means that a straight line drawn through the end of the vector b parallel to the applicate axis, once “entering” this cone, never “leaves” it again.

Generally speaking, after the transition from the basis $\beta^{(q)}$ to the basis $\beta^{(q+1)}$ we can re-form matrices $\Delta^{-1}\left(\beta^{(q+1)}\right)$, $\overline{\Delta}^{-1}\left(\beta^{(q+1)}\right)$ and, having calculated, draw conclusions about its optimality. However, given that $\overline{A}\left(\beta^{(q+1)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q+1)}\right)\overline{A}\beta^{(q+1)}$ differs from $\beta^{(q)}$ with just one column, from the point of view of computing technology, it seems rational to go directly from $\overline{A}\left(\beta^{(q)}\right)$ and $\overline{b}\left(\beta^{(q)}\right)$ to $\overline{A}\left(\beta^{(q+1)}\right)$ and $\overline{b}\left(\beta^{(q+1)}\right)$. The fact is that matrices of type $\overline{A}\left(\beta^{(q)}\right)$ the columns corresponding to the basis vectors consist of zeros, with the exception of one element equal to one. The position of this nonzero element is determined by the ordinal number of the base column in $N\left(\beta^{(q)}\right)$. Therefore, to obtain the matrix $\overline{A}\left(\beta^{(q+1)}\right)$ enough using linear operations on the rows of the matrix $\overline{A}\left(\beta^{(q)}\right)$ to bring its column, corresponding to the vector introduced into the basis, to the "basic" form.

To do this, apply *Jordan-Gauss transform* (the so-called method of complete exclusion). In this case, it consists in the fact that we must "earn" a unit in the place of the element $a_{rl}\left(\beta^{(q)}\right)$ (it is usually called *leading*) and zeros in the place of the remaining elements of the column $a_l\left(\beta^{(q)}\right)$ (recall that *l* is the number of the column entered into the basis, and r is the number of the row in the simplex table that defines the number of the column to be derived from the basis). The first is achieved by dividing the *r*-th row by the leading element, the second by adding the newly obtained *r*-th line, multiplied by a suitable coefficient, to the remaining rows of the matrix $\overline{A}(\beta^{(q)})$.

Formally, the result of performing this transformation on elements $\overline{A}(\beta^{(q)})$ and $\overline{b}(\beta^{(q)})$ can be expressed as follows:

$$a_{rj}(\beta^{(q+1)}) = \frac{a_{rj}(\beta^{(q)})}{a_{rl}(\beta^{(q)})}, j \in \overline{0,n}; \qquad (7.15)$$

$$b_r(\beta^{(q+1)}) = \frac{b_r(\beta^{(q)})}{a_{rl}(\beta^{(q)})}; \qquad (7.16)$$

$$a_{ij}(\beta^{(q+1)}) = a_{ij}(\beta^{(q)}) - a_{il}(\beta^{(q)})\frac{a_{rj}(\beta^{(q)})}{a_{rl}(\beta^{(q)})}, \text{where } i \in \overline{0,m}, i \neq r,$$

$$j \in \overline{1,n}; \qquad (7.17)$$

$$b_i(\beta^{(q+1)}) = b_i(\beta^{(q)}) - a_{il}(\beta^{(q)})\frac{b_r(\beta^{(q)})}{a_{rl}(\beta^{(q)})}, i \in \overline{0,m}, i \neq r. \quad (7.18)$$

Of particular note is the meaning of the elements of the vector $\overline{b}(\beta^{(q)})$. Its zero component $\overline{b}_0(\beta^{(q)})$ in accordance with the construction contains *value of the objective function* what she achieves in the current plan

$$f\left(x(\beta^{(q)})\right) = \overline{b}_0(\beta^{(q)}), \qquad (7.19)$$

and the remaining elements are *nonzero components* of this plan:

$$x_{N_{i(\beta^{(q)})}}(\beta^{(q)}) = b_i(\beta^{(q)}), i \in \overline{1,m}. \qquad (7.20)$$

The name of the method comes from the concept of simplex. Recall that a *m-simplex* is a convex polyhedron whose affine hull is an affine set of dimension *m*. A*ffin shell* sets are the smallest affine set in which a given set is contained. In this case, we can assume that the system of extended basis columns $a_{j_1}, a_{j_2}, \dots, a_{j_m}$ regarded as points in R^{m+1}, generates an (*m* – 1) -dimensional simplex in space R^{m+l}.

To conclude this section, we summarize the above questions and present a *simplex method algorithm diagram for solving the maximization problem*. It includes a once-performed 0-stage and a repeated finite number of times a 1-stage (standard iteration).

0 stage. Finding a valid baseline (see section 7.5.5). The result of the 0-stage is an acceptable basic plan $x\left(\beta^{(1)}\right)$, as well as the corresponding matrix $\overline{A}\left(\beta^{(1)}\right)$ and vector $\overline{b}\left(\beta^{(1)}\right)$, which will be used in the first iteration. We set the number of the current iteration q to 1 and go to the I-stage.

I stage. The standard iteration of the algorithm is executed for the next basic plan $x\left(\beta^{(q)}\right)$.

1 °. *Checking the optimality* of the current basic plan: the line of ratings $a_0\left(\beta^{(q)}\right)$ is reviewed. Two options are possible:

1'. $a_0\left(\beta^{(q)}\right) \geq 0$ – the plan corresponding to the current basis of the problem is ***optimal***. The computing process is complete. According to formulas (7.20) and (7.19), the optimal plan of the problem $x^* = x\left(\beta^{(q)}\right)$ and the value of the objective function $f(x^*) = f\left(x\left(\beta^{(q)}\right)\right)$ are written out.

1 ". In the line of ratings $a_0\left(\beta^{(q)}\right)$ there is at least one element $a_{0j}\left(\beta^{(q)}\right) <0$, i.e., having a negative rating. Therefore the plan$x\left(\beta^{(q)}\right)$ – ***not optimal***. A column with number l is selected that has a minimum (maximum in absolute value) negative rating

$$a_{0l}\left(\beta^{(q)}\right) = \min_{j:a_{01}\left(\beta^{(q)}\right)<0} \left\{a_{0j}\left(\beta^{(q)}\right)\right\}.$$

It is called *leading* and *must be entered into the next basis.* We pass to paragraph 2 ° of the algorithm.

2 °. *The definition of the column to be derived from the basis.* Leading column considered $a_l\left(\beta^{(q)}\right)$. Two options are possible:

2 '. For all $i \in \overline{1,m}$ $a_{0l}\left(\beta^{(q)}\right) \leq 0$.The conclusion about *unlimited target* functions and the computing process ends.

2 ". There is at least one line with a number $i \in \overline{1,m}$ for which $a_{0l}(\beta^{(q)}) > 0$. According to rule (7.14), the place r and the number $N_r(\beta^{(q)}) = j_r$ are determined for a column derived from *basis.* We pass to paragraph 3 ° of the algorithm.

3 °. *Recalculation of matrix elements* $\overline{A}$ and *the column* $\overline{b}$ relatively new basis. In accordance with formulas (7.15) – (7.18), we calculate the elements of the matrix of the problem $\overline{A}$ and vector constraints $\overline{b}$ relative to the basis $\beta^{(q+1)}$ that is obtained after introduction to the basis $\beta^{(q)}$ the column a_l and outputting a column a_r from it. We assume the current iteration number $q = q + 1$ and go to the first paragraph of the algorithm.

Note. If the objective function requires finding the minimum value, then the criterion for the optimality of the problem is the absence of positive estimates to $a_{0j}(\beta^{(q)})$ in the line of the objective function for all $j = 12,...,n.$

7.5.2. Tabular implementation of the simplex method.

In terms of security the rationality and clarity of the computing process, the execution of the simplex method algorithm is conveniently arranged in the form of a sequence of tables. Different sources provide different modifications of simplex tables that differ from each other in the arrangement of individual elements. However, they are all based on the same principles. Let us dwell on the structure of the table shown in Fig. thirteen.

$$T^{(q)} = \begin{array}{|c|c|c|} \hline & B_0(\beta^{(q)}) & a_0(\beta^{(q)}) \\ N(\beta^{(q)}) & b(\beta^{(q)}) & A(\beta^{(q)}) \\ \hline \end{array}$$

Fig. 13

Simplex table $T^{(q)}$, depicted in fig. 13 corresponds to the permissible CTLP basis obtained at the q-th iteration. The column $N(\beta^{(q)})$ contains the numbers of the base columns (in the order in which they enter the basis), the column $b(\beta^{(q)})$ –

components of the constraint vector with respect to the current basis $\beta^{(q)}, A(\beta^{(q)})$ – components of the matrix of the problem with respect to the current basis $\beta^{(q)}$. Finally, the row $a_0(\beta^{(q)})$ contains the current column estimates, and the cell $b_0\left(\beta^{(q)}\right)$ contains the objective function value achieved for the current plan.

Of course, it should be added that tabular modification of the simplex method is of great practical importance not only as a convenient form of organizing a manual account, but as a basis for implementing this algorithm in the framework of computer software.

7.5.3. CTLP solution example by simplex method.

Consider a concrete example of the process of solving CTLP tabular simplex method. Let the canonical LP problem be given:

$$f(x) = 50x_1 - 10x_2 + 6x_3 + 40x_4 - 30x_5 \to \text{max}. \qquad (7.21)$$

$$\begin{cases} x_1 + x_4 + x_5 = 12, \\ 2x_1 + x_2 + x_4 = 14, \\ -2x_1 + 3x_3 - x_4 = 17, \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0.$$

As you can see, the columns of the matrix $A = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ -2 & 0 & 3 & -1 & 0 \end{pmatrix}$ of the constraint system with numbers {5, 2, 3} are linearly independent. And you can get the decomposition of the constraint vector $b = \begin{pmatrix} 12 \\ 14 \\ 17 \end{pmatrix}$ with positive coefficients in these columns:

$$b = \begin{pmatrix} 12 \\ 14 \\ 17 \end{pmatrix} = 12\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 14\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \frac{17}{3}\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}.$$

The latter means that the columns {5, 2, 3} form an admissible basis from which the solution of the problem can begin. From the columns included in the basis, taking into account the zero elements, a matrix $\overline{\Delta}(\beta^{(1)})$ is formed and the opposite with respect to it $\overline{\Delta}^{-1}(\beta^{(1)})$:

$$\overline{\Delta}\left(\beta^{(1)}\right) = \begin{pmatrix} -1 & -30 & -10 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix},$$

$$\overline{\Delta}^{-1}\left(\beta^{(1)}\right) = \begin{pmatrix} -1 & -30 & -10 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/3 \end{pmatrix}.$$

Using them, according to the formulas $\overline{A}\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{A}$ and $\overline{b}\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{b}$ we obtain:

$$\overline{A}\left(\beta^{(1)}\right) = \overline{\Delta}^{-1}\left(\beta^{(1)}\right)\overline{A} =$$

$$= \begin{pmatrix} -1 & -30 & -10 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/3 \end{pmatrix} \times\times \begin{pmatrix} 50 & -10 & 6 & 40 & -30 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ -2 & 0 & 3 & -1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -104 & 0 & 0 & -82 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ -2/3 & 0 & 1 & -1/3 & 0 \end{pmatrix};$$

$$\overline{b}\left(\beta^{(1)}\right) = \overline{\Delta}^{-1}\left(\beta^{(1)}\right) \times \overline{b} = \begin{pmatrix} -1 & -30 & -10 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 12 \\ 14 \\ 17 \end{pmatrix} == \begin{pmatrix} -466 \\ 12 \\ 14 \\ 17/3 \end{pmatrix}.$$

Using the obtained values of $\overline{A}\left(\beta^{(1)}\right)$ and $\overline{b}\left(\beta^{(1)}\right)$, we fill in the simplex table $T^{(1)}$ corresponding to the first iteration (q = 1)

$T^{(1)} =$

	–466	–104	0	0	–82	0
5	12	1	0	0	1	1
2	14	2	1	0	1	0
3	17/3	–2/3	0	1	–1/3	0

Since the line of estimates $a_0\left(\beta^{(1)}\right)$ (underlined by a dashed line) in the first and fourth columns contains negative elements элементы $\left(a_{01}\left(\beta^{(1)}\right)\right.$ = – 104, $a_{04}\left(\beta^{(1)}\right)$ = – 82) the plan $x\left(\beta^{(1)}\right)$ = (0, 14, 17/3, 0, 12) is not optimal, and the value of the objective function $f\left(x\left(\beta^{(1)}\right)\right)$ = – 466 can be improved. Following the recommendation of paragraph 1"of the simplex method algorithm, we set the number of the column introduced into the next basis (since l = 1 (т. к. |– 104| > |– 82|). Consider the *leading* column (highlighted by a frame)

$$a_{01}\left(\beta^{(1)}\right) = \begin{pmatrix} 1 \\ 2 \\ -2/3 \end{pmatrix}.$$

It contains two positive elements. Using the recommendation of paragraph 2 of the algorithm, we get λ_1 = 12/1 = 12, λ_2 = 14/2 = 7, and therefore r = 2. Therefore, the column with number $N_2\left(\beta^{(1)}\right)$ should be taken out of the basis. Thus, we get the next admissible basis of the problem $N\left(\beta^{(2)}\right) = \{5,\ 1,\ 3\}$ and go to the iteration q = 2.

$$\overline{\Delta}\left(\beta^{(2)}\right) = \begin{pmatrix} -1 & -30 & 50 & 6 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -2 & 3 \end{pmatrix},$$

$$\overline{\Delta}^{-1}\left(\beta^{(2)}\right) = \begin{pmatrix} -1 & -30 & 42 & 2 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1/3 & 1/3 \end{pmatrix}.$$

Using them, according to the formulas $\overline{A}\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{A}$ and $\overline{b}\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{b}$, we obtain:

$$\overline{b}\left(\beta^{(2)}\right) = \overline{\Delta}^{-1}\left(\beta^{(2)}\right) \times \overline{b} = \begin{pmatrix} -1 & -30 & 42 & 2 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1/3 & 1/3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 12 \\ 14 \\ 17 \end{pmatrix} =$$

$$= \begin{pmatrix} 262 \\ 5 \\ 7 \\ 31/3 \end{pmatrix};$$

$$\overline{A}\left(\beta^{(2)}\right) = \overline{\Delta}^{-1}\left(\beta^{(2)}\right) \overline{A} = \begin{pmatrix} -1 & -30 & 42 & 2 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \times$$

$$\times \begin{pmatrix} 50 & -10 & 6 & 40 & -30 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ -2 & 0 & 3 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 52 & 0 & -30 & 0 \\ 0 & -1/2 & 0 & 1/2 & 1 \\ 1 & 1/2 & 0 & 1/2 & 0 \\ 0 & 1/3 & 1 & 0 & 0 \end{pmatrix}.$$

Using the obtained values $\overline{A}\left(\beta^{(2)}\right)$ and $\overline{b}\left(\beta^{(2)}\right)$ we fill in the simplex table $T^{(2)}$ corresponding to the second iteration (q = 2)

$T^{(2)}$ =

	262	0	52	0	- 30	0
5	5	0	–1/2	0	1/2	1
1	7	1	1/2	0	1/2	0
3	31/3	0	1/3	1	0	0

In the course of the second iteration, the input column l = 4 (marked with a frame), the output r = 5, is again determined. Passing to the iteration q = 3 for the basis $N\left(\beta^{(2)}\right)$= {4, 1, 3}, we have:

$$\overline{\Delta}\left(\beta^{(3)}\right) = \begin{pmatrix} -1 & 40 & 50 & 6 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 3 \end{pmatrix},$$

$$\overline{\Delta}^{-1}\left(\beta^{(3)}\right) = \begin{pmatrix} -1 & 30 & 12 & 2 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1/3 & 1/3 \end{pmatrix}.$$

Using them, according to the formulas $\overline{A}\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{A}$ and $\overline{b}\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{b}$, we obtain:

$$\overline{A}\left(\beta^{(3)}\right) = \overline{\Delta}^{-1}\left(\beta^{(3)}\right)$$

$$= \begin{pmatrix} -1 & 30 & 12 & 2 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1/3 & 1/3 \end{pmatrix} \times \begin{pmatrix} 50 & -10 & 6 & 40 & -30 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ -2 & 0 & 3 & -1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 22 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 & -1 \\ 0 & 1/3 & 1 & 0 & 0 \end{pmatrix};$$

$$\overline{b}\left(\beta^{(3)}\right) = \overline{\Delta}^{-1}\left(\beta^{(3)}\right) \times \overline{b} = \begin{pmatrix} -1 & 30 & 12 & 2 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1/3 & 1/3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 12 \\ 14 \\ 17 \end{pmatrix} = \begin{pmatrix} 562 \\ 10 \\ 2 \\ 31/3 \end{pmatrix}.$$

Using the obtained values of $\overline{A}\left(\beta^{(3)}\right)$ and $\overline{b}\left(\beta^{(3)}\right)$, we fill in the simplex table $T^{(3)}$ corresponding to the third iteration (q = 3)

$T^{(3)} =$

	562	0	22	0	0	0
4	10	0	–1	0	1	2
1	2	1	1	0	0	–1
3	31/3	0	1/3	1	0	0

As can be seen from the $T^{(3)}$ row of estimates contains only non-negative values, therefore the achieved basis $N(\beta^{(3)})$ = {4, 1, 3} is optimal. As a result, we get that the vector x * = (2, 0, 31/3, 10, 0) is the optimal plan (maximum point) of the problem, the maximum value of the objective function is f * = f (x *) = 562, and the solution CTLP has the form ((2, 0, 31/3, 10, 0); 362).

7.5.4. Convergence of the simplex method. Degeneracy in LP problems. The most important property of any computational algorithm is *convergence*, i.e., the possibility of obtaining the desired results (with a given accuracy) in a finite number of steps (iterations) during its application.

It is easy to see that problems with the convergence of the simplex method can potentially arise at the stage of choosing the value of *r* (item 2 ") in the case when the same minimum values of the ratio

$$\lambda_i = \frac{b_i(\beta^{(q)})}{a_{il}(\beta^{(q)})}, \text{где } i \in \{\overline{1,m} | a_{il}(\beta^{(q)}) > 0\}.$$

will be reached for several rows of the table $T^{(q)}$ at the same time. Then at the next iteration, the column $b(\beta^{(q+l)})$ will contain null elements. Recall that

► *admissible basic plan of the canonical LP problem corresponding to the basis $\beta^{(q)}$, called* ***degenerate****, if some of its basis components are equal to zero, i.e., the vector $b(\beta^{(q)})$ contains zero elements.*

A LP problem that has degenerate plans is called ***degenerate***. When we "enter" a degenerate plan, we actually get the decomposition of column *b* in the system from less than *m* columns a_j and therefore, we lose the opportunity to correctly determine the input of which column in the basis will lead to an increase in the value of the objective function. Such situations, ultimately, can lead to a looping of the computational process, i.e., an endless enumeration of the same bases.

From the point of view of the second geometric interpretation of the TLP, degeneracy means "falling" of a line passing through the top of the vector *b* parallel to the applicate axis into the edge of the cone stretched over the system of extended vectors of the current basis. So, in the case depicted in fig. 14, this line falls into the edge defined by the vector $\overline{a}^{j_2}$, i.e., despite the fact that the current basis contains columns $\overline{a}^{j_1}$, and $\overline{a}^{j_2}$, only one is enough to represent the vector *b* $\overline{a}^{j_2}$.

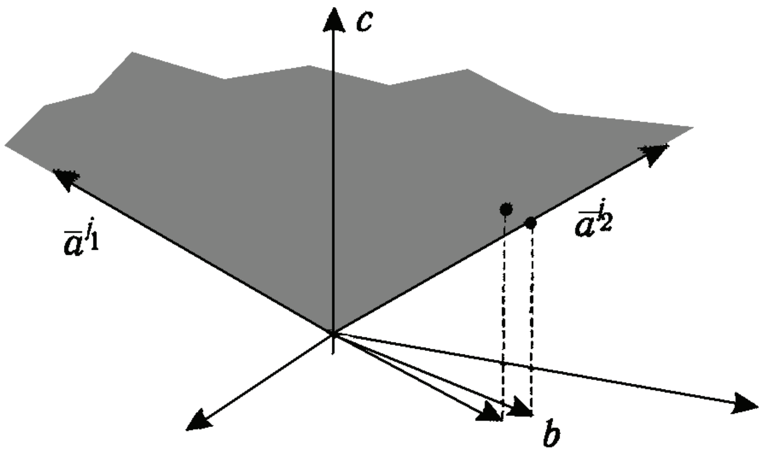


Fig. 14

This interpretation also implies the idea of a method for solving degenerate LP problems, called *perturbation method*: upon reaching the degenerate plan, a slight shift of the vector b occurs and the degeneracy (as can be seen from Fig. 14) is eliminated.

However, it must be said that the looping problem discussed here for most practically significant tasks is quite rare and unlikely. Moreover, it is very often resolved due to rounding errors during computer calculations.

7.5.5. Finding the initial acceptable baseline in LP problems.

In the example considered above, the initial basic plan needed to start the calculations using the simplex method was selected due to the features of the conditions matrix. Indeed, this matrix already contained the required number of “almost basic” columns. Obviously, for the vast majority of LP problems, it is impossible to immediately and explicitly indicate the initial acceptable baseline plan in this way. Generally speaking, there are various methods for solving this problem. We will focus on one of them, called the method of *minimizing residuals*. Its strength, of course, is versatility. Although, in some special cases, it may be too cumbersome.

The idea of the method of minimizing residuals consists in constructing the corresponding auxiliary problem for which you can explicitly indicate the initial basic plan, and solving it using the simplex method procedure.

Without loss of generality, we can assume that the vector of constraints in the original problem is non-negative, i.e. $b_i \geq 0$, $i \in \overline{1,m}$. Then for CTLP maximizing the function

$$f(x) = c_1x_1 + c_2x_2 + \cdots + c_nx_n \to \max \qquad (7.22)$$

on the set $D \subset R^n$ defined by the constraints

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m, \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$$

auxiliary task is being constructed

$$\tilde{f}(x) = 0x_1 + 0x_2 + \cdots + 0x_n - 1x_{n+1} - 1x_{n+2} - \cdots - 1x_{n+m} \to \max \qquad (7.23)$$

$$\tilde{D} == \left\{ x \in R^{n+m} \middle| \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + 1x_{n+1} + 0x_{n+2} + \cdots + 0x_{n+m} = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + 0x_{n+1} + 1x_{n+2} + \cdots + 0x_{n+m} = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + 0x_{n+1} + 0x_{n+2} + \cdots + 1x_{n+m} = b_m, \\ x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0, x_{n+1} \geq 0, \ldots, x_{n+m} \geq 0. \end{cases} \right\}.$$

As can be seen from (7.22), (7.23), the problem $(\tilde{D}, \tilde{f})$ differs from the problem (D, f) by the matrix obtained by adding a unit matrix to the original matrix A, which corresponds to additional (dummy) variables $x_{n+1}, x_{n+2}, \ldots, x_{n+m}$. The coefficients (–1) correspond to these variables (in fact, they are called the *residuals*) in the objective function, and the zero coefficients correspond to the variables $x_1, x_2, \ldots, x_n$ that are common to both tasks. An essential feature is the presence in it of an obvious initial basis formed by the added columns, and an appropriate plan with basic components $x_{n+i} = b_i \geq 0, i \in \overline{1,m}$. Due to the structure of the objective function $\tilde{f}$, during the search for its maximum by the simplex method procedure, dummy variables x_{n+i} should preferably be minimized to zero, which can be achieved by sequentially translating them into non-basic components of the plan. If in the optimal plan $\tilde{x}$ obtained as a result of solving the auxiliary problem, the last *m* components (i.e., residuals) are equal to zero, then its initial n components satisfy the system of constraints defining region *D* and can easily be converted to the starting one by simply discarding the residuals admissible plan of the main problem (*D, f*). Moreover, all the rows of the simplex table obtained at the last iteration of the auxiliary task (except for the zero one) can be transferred to the first table of the main task. The elements of the zero row for the newly formed simplex table are calculated by the formulas:

$$a_{0j}\left(\beta^{(1)}\right) = -c_j + c\left(\beta^{(*)}\right)a_j\left(\beta^{(*)}\right), \qquad (7.24)$$

$$b_0\left(\beta^{(1)}\right) = c\left(\beta^{(*)}\right)b\left(\beta^{(*)}\right), \qquad (7.25)$$

where $\left(\beta^{(*)}\right)$ is the basis obtained at the last iteration of the auxiliary problem.

In the case when the optimal plan $\tilde{x}$ the auxiliary problem still contains non-zero dummy components (i.e. $\tilde{f}(\tilde{x}) < 0$), we conclude that there are no feasible plans for the original problem (*D, f*), i.e., $D = \emptyset$.

7.6. Modified simplex method.

7.6.1. Computational scheme based on the inverse matrix transformation.

By analyzing the computational procedure of the simplex method from the standpoint of laboriousness estimation, it is easy to notice that the most critical in this regard is the stage of recalculation of the values of *A* and *b* during the transition from one basic plan to another (Sec. 3° algorithm). However, in the case when the number of constraints of the problem *m* clearly fewer variables *n,* significant “savings” can be achieved by performing the Jordan – Gauss transformation at the next iteration *q* not over the matrix $\overline{A}\left(\beta^{(q)}\right)$, but over the matrix $\overline{\Delta}^{-1}\left(\beta^{(q)}\right)$. This takes into account the fact that, if necessary, using the formula $\overline{A}\left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{A}$, you can always get $\overline{A}\left(\beta^{(q)}\right)$ by $\overline{\Delta}^{-1}\left(\beta^{(q)}\right)$. Moreover, to perform the actions of the simplex procedure described above, we actually did not need a matrix $\overline{A}\left(\beta^{(q)}\right)$ entirely. In reality, it used only the row of ratings $a_0(\beta^{(q)})$ and the leading column $a_l(\beta^{(q)})$. These considerations form the basis of the computational scheme of the simplex method based on the inverse matrix transformation, which is also called *modified simplex-method.* This algorithm was first proposed in 1951 in the works of L.V. Kantorovich.

The computational scheme of the modified simplex method corresponds to the system of tables T_1 and $T_2^{(q)}$. Table T_1 (fig. 15) is common to all iterations and serves to obtain a string of estimates of the current basic plan $a_0(\beta^{(q)})$. If denoted by $\delta_i(\beta^{(q)})$, $i \in \overline{0,m}$ rows of the matrix $\overline{\Delta}^{-1}(\beta^{(q)})$, it follows from $\overline{A}(\beta^{(q)}) = \overline{\Delta}^{-1}(\beta^{(q)})\,\overline{A}$, in particular, that

$$a_0(\beta^{(q)}) = \delta_0(\beta^{(q)})\overline{A}. \qquad (7.26)$$

As seen from Fig. 15, T_1 consists of three blocks:

- *the center contains a matrix* $\overline{A}$;
- *in the left block of the table, at each iteration, zero rows of the matrix* $\overline{\Delta}^{-1}(\beta^{(q)})$ *are added for current basis*;
- *the lower block, located under the matrix* $\overline{A}$*, at each iteration is supplemented by a line of estimates of the current plan, calculated by the formula* (7.26).

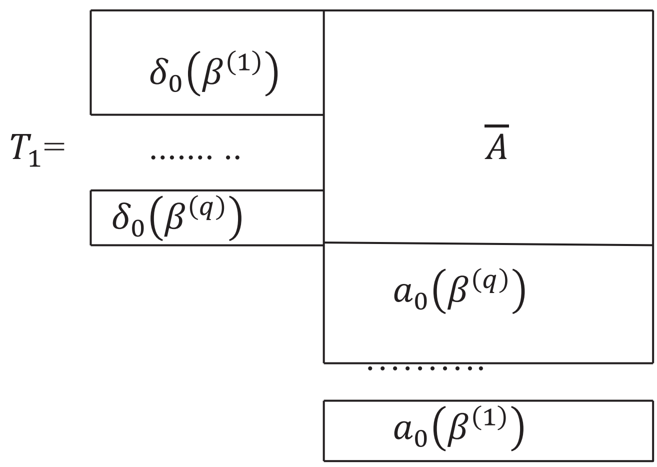


Fig. 15

The simplex table $T_2^{(q)}$ shown in Fig.16, corresponds to the permissible CTLP basis $\beta^{(q)}$ obtained on q-th iterations.

$T_2^{(q)} =$

	$b_0(\beta^{(q)})$			
$N(\beta^{(q)})$	$b(\beta^{(q)})$	$\overline{\Delta}^{-1}(\beta^{(q)})$	$\overline{a}_l$	$\overline{a}_l(\beta^{(q)})$

Fig. 16

Column $N(\beta^{(q)})$ contains the numbers of the base columns (in the sequence of occurrence in the basis); column $b(\beta^{(q)})$ – component of the constraint vector relative to the current basis $\beta^{(q)}$; $\overline{\Delta}^{-1}(\beta^{(q)})$ – matrix inverse to the matrix of extended columns of the current basis $\beta^{(q)}$; count $\overline{a}_l$ – contains an extended vector of conditions introduced into the basis at the current iteration, and the next graph contains the coordinates $\overline{a}_l(\beta^{(q)})$ of the same column in the current basis $\beta^{(q)}$.

By analogy with section 7.5.1, we describe the formal scheme of the modified simplex method algorithm.

0 stage. Finding a valid baseline.

1. To find an acceptable basis, the residual minimization method, discussed in Section 7.5.5, can be applied. Moreover, to solve the auxiliary problem, the procedure of the modified simplex method is used. As a result of the 0-stage, we get an acceptable basic plan $x(\beta^{(q)})$ and the corresponding matrix $\overline{\Delta}^{-1}(\beta^{(1)})$ and vector $\overline{b}(\beta^{(1)})$.

2. We fill in the central part of the table T_1 containing the matrix $\overline{A}$.

3. Content matrix $\overline{\Delta}^{-1}(\beta^{(1)})$ and the vector $\overline{b}(\beta^{(1)})$ obtained at the stage of searching for an acceptable baseline is transferred to the table$T_2^{(1)}$.

4. We put the number of the current iteration q equal to 1 and go to the I-stage.

I stage. The standard iteration of the algorithm is executed for the next basic plan $x(\beta^{(q)})$.

1 °. *Checking the optimality of the current baseline.* Table zero row content $T_2^{(q)} - \delta_0\left(\beta^{(q)}\right)$corresponds to the corresponding column of the table T_1. According to the formula (7.26), we calculate and fill the line $a_0\left(\beta^{(q)}\right)$. Two options are possible:

1'. $a_0\left(\beta^{(q)}\right) \geq 0$ – the plan corresponding to the current basis of the problem is ***optimal***. The computing process is complete. According to formulas (7.19) and (7.18), the optimal plan of the problem is written $x^* = x\left(\beta^{(q)}\right)$ and the optimal value of the objective function $f\ (x^*) = f\left(x\left(\beta^{(q)}\right)\right)$.

1". In the line of ratings $a_0\left(\beta^{(q)}\right)$ there is at least one element $a_{0j}\left(\beta^{(q)}\right) < 0$, i.e., having a negative rating. Therefore the plan $x_0\left(\beta^{(q)}\right)$ ***not optimal.*** The number l corresponding to the element with the minimum (maximum in absolute value) negative rating is selected:

$$a_{0l}\left(\beta^{(q)}\right) = \min_{j:a_{01}\left(\beta^{(q)}\right)<0} \left\{a_{01}\left(\beta^{(q)}\right)\right\}.$$

The l-th column of the matrix $\overline{A}$ becomes *leading* and should be introduced into the next basis. We pass to paragraph 2° of the algorithm.

2°. *The definition of the column to be derived from the basis*. Rewrite the leading column $\overline{a}_l$ from the table T_1 to the current table $T_2^{(q)}$. Using the formula $\overline{a}_l\ \left(\beta^{(q)}\right) = \overline{\Delta}^{-1}\left(\beta^{(q)}\right)\overline{a}_l$, fill in the corresponding column in the table $T_2^{(q)}$. Two options are possible:

2 '. For all $i \in \overline{1,m}$ $a_{il}\left(\beta^{(q)}\right) \leq 0$. The conclusion is made that the *objective function is unlimited* and the computational process is completed.

2 ". There is at least one index $i \in \overline{1,m}$ for which $a_{il}\left(\beta^{(q)}\right) > 0$. According to rule (7.13), the place r and the number are determined $N_r\left(\beta^{(q)}\right) = j_r$ for a column derived from *basis.* We pass to paragraph 3° of the algorithm.

3°. *Recalculation with respect to the new basis of elements of column b and matrix* $\overline{\Delta}^{-1}$. Transition to a new basis $\beta^{(q+1)}$, which is obtained by introducing $\beta^{(q)}$ column a_r and the derivation of a column a_l from it, is carried out according to formulas similar to formulas (7.15) – (7.18). They have the form:

$$b_r\left(\beta^{(q+1)}\right) = \frac{b_r\left(\beta^{(q)}\right)}{a_{rl}\left(\beta^{(q)}\right)}; \qquad (7.27)$$

$$\left(\beta^{(q+1)}\right) = \frac{\delta_{rj}\left(\beta^{(q)}\right)}{a_{rj}\left(\beta^{(q)}\right)}, j \in \overline{0, m.}; \qquad (7.28)$$

$$b_i\left(\beta^{(q+1)}\right) = b_i\left(\beta^{(q)}\right) - b_r\left(\beta^{(q)}\right)\frac{a_{ij}\left(\beta^{(q)}\right)}{a_{rl}\left(\beta^{(q)}\right)}, i \in \overline{0, m}, i \neq r; \quad (7.29)$$

$$\delta_{ij}\left(\beta^{(q+1)}\right) = \delta_{ij}\left(\beta^{(q)}\right) - \delta_{rj}\left(\beta^{(q)}\right)\frac{a_{il}\left(\beta^{(q)}\right)}{a_{rl}\left(\beta^{(q)}\right)}, i \in \overline{1, m},$$

$$i \neq r, j \in \overline{1, m}. \qquad (7.30)$$

We put the number of the current iteration q equal to $q + 1$ and go to the first paragraph of the algorithm.

7.6.2. An example of solving TLP by the modified simplex method.

We give a solution to the previously considered problem (7.21) based on the use of the procedure modified simplex method. By analogy with Section 7.5.3, it begins by choosing the obvious initial basis formed by the columns {5, 2, 3}. For him have already been calculated $\overline{\Delta}^{-1}\left(\beta^{(q)}\right)$ and $\overline{b}\left(\beta^{(q)}\right)$ therefore, filling in the initial tables T_1 and $T_2^{(1)}$ It is not difficult (Fig. 17).

In the first iteration, we rewrite the null string $\delta_0\left(\beta^{(0)}\right) = (-1, -30, -10, 2)$ from the table $T_2^{(1)}$ at T_1 and, multiplying it by the matrix $\overline{A}$, we get a row of estimates $a_0\left(\beta^{(1)}\right) = \delta_0\left(\beta^{(1)}\right)\overline{A} = (-104, 0, 0, -82, 0)$. As $a_0\left(\beta^{(1)}\right)$ contains negative elements, then we conclude that the plan corresponding to the basis $\beta^{(1)}$ is not optimal, and choosing the smallest negative estimate (–104), we get the number of the column introduced into the basis, $l = 1$.

$T_1 =$

–1	–30	–10	2	50	–10	6	40	–30
–1	–30	42	2	1	0	0	1	1
–1	30	12	2	2	1	0	1	0
				–2	0	3	–1	0
				–104	0	0	–82	0
				0	52	0	–30	0
				0	22	0	0	0

$T_2^{(1)} =$

	–466	–1	–30	–10	2	50	–104	
5	12	0	1	0	0	1	1	
2	14	0	0	1	0		2	2
3	17/3	0	0	0	1/3	–2	–2/3	

Fig. 17

Rewrite the column $\overline{a}_1 = \begin{pmatrix} 50 \\ 1 \\ 2 \\ -2 \end{pmatrix}$ from table T_1 to $T_2^{(1)}$ and recalculate its coordinates relative to the current basis, i.e., multiply the matrix $\overline{\Delta}^{-1}\left(\beta^{(q)}\right)$ located in the table $T_2^{(1)}$ on the left by $\overline{a}_1$.

After filling in the table $T_2^{(1)}$ with the data on the column entered into the new basis, one can go on to determine the number of the displayed column. This procedure is carried out in complete analogy with the usual simplex method. Having examined the relationship of elements $\overline{b}\left(\beta^{(1)}\right)$ and $a_{il}\left(\beta^{(1)}\right)$ for $\left\{i \in \overline{1,m} \middle| a_{il}\left(\beta^{(1)}\right) > 0\right\}$ and determining the minimum of them (λ_2 =14/2), we find that r = 2. Therefore, the column with the number $N_2\left(\beta^{(q)}\right)$ = 2 must be derived from the basis. Thus, we obtain the next admissible basis of the problem with $N\left(\beta^{(2)}\right)$ = {5, 1, 3}. Applying formulas (7.28) – (7.30), we pass to the simplex table corresponding to the second iteration $T_2^{(2)}$ (Fig. 18), and we set the index of the current iteration $q = 2$.

$T_2^{(2)} =$

	262	–1	–30	42	2	40	–30
5	5	0	1	–1/2	0	1	1/2
1	7	0	0	1/2	0	1	1/2
3	31/3	0	0	1/3	1/3	–1	0

Fig. 18

Repeating the same steps (they can easily be traced in the tables given here $T_2^{(2)}$ and $T_2^{(3)}$), at the third iteration we get the optimal plan of the problem and the optimal value of the objective function, which are extracted from the second column of the table $T_2^{(3)}$ (fig. 19). It is easy to see that in the process of solving we "walked" along the same sequence of admissible basic plans that was found in section 2.5.3.

$T_2^{(3)} =$

	562	–1	30	12	2
4	10	0	2	–1	0
1	2	0	–1	1	0
3	31/3	0	0	1/3	1/3

Fig. 19

In conclusion, we emphasize that, due to the above advantages, it is the modified simplex method that is actually used in software designed to solve canonical linear programming problems. A significant advantage of the modified simplex method is that it allows you to simultaneously find the optimal plans for both direct and dual tasks.

7.7. Duality in linear programming problems.

7.7.1. The concept of the dual LP problem and the general scheme of its construction.

Every common task (D, f) linear programming, called *direct, or source*, is closely related to another general task (D^*, f^*), which is called ***dual.***

The mathematical models of these general problems (D, f) and (D^*, f^*) LP have the following form.

Direct (initial) task:

$$f(x) = c_1x_1 + c_2x_2 + \cdots + c_jx_j + c_{j+1}x_{j+1} + \cdots + c_nx_n \to max,$$

$$x \in D, \qquad (7.31)$$

where D is determined by a system of equations and inequalities:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq b_i, \\ a_{(i+1)1}x_1 + a_{(i+1)2}x_2 + \cdots + a_{(i+1)n}x_n = b_{i+1}, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m, \\ x_1 \geq 0, x_2 \geq 0, \dots, x_j \geq 0, \end{cases}$$

We assume that in this system the first i constraints are inequalities, and the subsequent ones are equations. This can always be achieved by reordering the inequalities and the equalities of the system.

The dual task:

$$f^*(y) = b_1y_1 + \cdots + b_iy_i + b_{i+1}y_{i+1} + \cdots + b_my_m \to min,$$

$$y \in D^*, \qquad (7.32)$$

where D^* is determined by a system of equations and inequalities:

$$\begin{cases} a_{11}y_1 + a_{21}y_2 + \cdots + a_{m1}y_m \geq c_1, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ a_{1j}y_1 + a_{2j}y_2 + \cdots + a_{mj}y_m \geq c_j, \\ a_{1(j+1)}x_1 + a_{2(j+1)}x_2 + \cdots + a_{m(j+1)}y_m = c_{j+1}, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{1n}y_1 + a_{2n}y_2 + \cdots + a_{mn}y_m = c_n, \\ y_1 \geq 0, y_2 \geq 0, \dots, y_i \geq 0, \end{cases}$$

The rules for constructing a task dual to the general LP problem are graphically represented by the circuit shown in Fig. 20.

c_1	…	c_j	c_{j+1}	…	c_n

$\rightarrow max$

min

↑

y_1	a_{11}	…	a_{1j}	$a_{1(j+1)}$	…	a_{1n}	b_1
…	…	…	…	…	…	…	…
y_i	a_{i1}	…	a_{ij}	$a_{i(j+1)}$	…	a_{in}	b_i
y_{i+1}	$a_{(i+1)1}$	…	$a_{(i+1)j}$	$a_{(i+1)(j+1)}$	…	$a_{(i+1)n}$	b_{i+1}
…	…	…	…	…	…	…	…
y_m	a_{m1}		a_{mj}	$a_{m(j+1)}$		a_{mn}	b_m

x_1	…	x_j	x_{j+1}	…	x_n

Fig. 20

From the above definition follows an important property - symmetry of the duality relation, i.e., a problem dual to dual $((D^*)^*, (f^*)^*)$coincides with the direct (initial) task (D, f):

$$((D^*)^*, (f^*)^*) = (D, f).$$

Thus, it makes sense to talk about a pair of mutually dual tasks, in the construction of which The following conditions must be observed:

1. If the direct problem is solved at the maximum, then the dual task is solved at the minimum; if the direct problem is solved at a minimum, then the dual one - at a maximum.

2. The row vector of the coefficients of the objective function c and the column vector of constraints b are interchanged. In the maximum problem, the inequality constraints have the form $\leq$, and in the minimum problem -.$\geq$

3. The constraint matrix of problem A is transposed.

4. The set of indices of variables on which the condition of non-negativity in the direct problem is imposed (for example, $x_j \geq 0$ or $y_i \geq 0$), determine the numbers of constraints in the form of inequalities in the dual problem ($a_j \cdot y \leq c_j$ or $a_i \cdot x \leq b_i$), otherwise, as an equality constraint.

5. The set of constraint numbers in the form of inequalities in the direct problem (for example, $a_i \cdot x \leq b_i$ or $a_j \cdot y \leq c_j$) determine the set of indices of variables on which the condition of non-negativity is imposed in the dual problem ($y_i \geq 0$ or $x_j \geq 0$). Accordingly, if any restriction of the direct problem is written as equality then the condition of non-negativeness is not imposed on the corresponding variable of the dual problem.

Consider the process of constructing a dual problem on a concrete example. Let the general LP problem (*D, f*) be given:

$$f(x) = 5x_1 - 2x_2 + 7x_3 + 4x_4 - 3x_5 \rightarrow max, x \in D,$$

$$D = \left\{ x \in R^5 \left| \begin{array}{c} 4x_1 + x_2 - x_3 + x_4 \leq 2, \\ 5x_2 + x_3 - 6x_4 + 2x_5 = 4, \\ 2x_1 + 3x_2 + 6x_3 + x_4 - 3x_5 \leq 5, \\ x_2 \geq 0, \qquad x_5 \geq 0. \end{array} \right. \right\}$$

Then the problem (*D*, f**) will be dual to it:

$$f^*(y) = 2y_1 + 4y_2 + 5y_3 \rightarrow min, y \in D^*,$$

$$D^* = \left\{ y \in R^3 \left| \begin{array}{c} 4y_1 + 2y_3 = 5, \\ y_1 + 5y_2 + 3y_3 \geq -2, \\ -y_1 + y_2 + 6y_3 = 7, \\ y_1 - 6y_2 + y_3 = 4, \\ 2y_2 - 3y_3 \geq -3, \\ y_1 \geq 0, y_3 \geq 0. \end{array} \right. \right\}$$

In matrix form, a pair of dual general LP problems (provided that all constraints are in the form of inequalities $\leq$ and all variables x_j are non-negative) can be briefly written as:

$$f(x) = c \cdot x \rightarrow max, \qquad D = \{x \in R^n | Ax \leq b, x \geq 0\}$$

and

$$f^*(y) = y \cdot b \rightarrow min, \ D^* = \{y \in R^m | yA \geq c, y \geq 0\}.$$

Here the constraint matrix *A* has dimension $m \times n$.

If the direct problem (*D, f*) and the dual problem (D^*, f^*) are given canonically, then in matrix form they respectively take the form:

$$f(x) = c \cdot x \to max, \qquad D = \{x \in R^n | Ax = b, x \geq 0\};$$

$$f^*(y) = y \cdot b \to min, \; D^* = \{y \in R^m | yA \geq c\}.$$

The solutions of dual problems are interconnected and this interconnection is reflected in duality theorems.

7.7.2. Duality theorems and their application.

The fundamental properties that establish the relationship between the solutions of the direct and dual LP problems can be formulated in the form of the statements below. They are usually called *duality theorems*.

First duality theorem. If one of the dual problems has an optimal solution $\tilde{x}$, then the other problem has an optimal solution $\tilde{y}$, and the extreme values of the objective functions coincide: $f(\tilde{x}) = f^*(\tilde{y})$.

The converse is also true. If for some admissible plans *x* and *y* of mutually dual problems (*D, f*) and (*D*, f**) the equality $f(\tilde{x}) = f^*(\tilde{y})$ fulfilled, then $\tilde{x}$ and $\tilde{y}$ are optimal plans for these tasks.

Second duality theorem. If one of the dual problems is unsolvable due to the unboundedness of the objective function on the set of feasible solutions, then the constraint system of the other problem is contradictory, and it has no feasible plans.

Third duality theorem. Let $x^* = (x_1^*, x_2^*, \dots, x_n^*)$ and $y^* = (y_1^*, y_2^*, \dots, y_m^*)$ be the optimal plans of the canonical problem (*D, f*) and the dual problem (*D*, f**) with respect to it. If the *j*-th component of the plan *x** is strictly positive ($x_j^* > 0$), then the corresponding *j*-th restriction of the dual problem is fulfilled as equality: $a_{1j}y_1^* + a_{2j}y_2^* + \dots + a_{mj}y_m^* = c_j$; if the *j*-th component of the plan *x** has a zero value ($x_j^* = 0$), then the *j*-th restriction of the dual problem is fulfilled as an inequality:

$$a_{1j}y_1^* + a_{2j}y_2^* + \dots + a_{mj}y_m^* \geq c_j.$$

Evidence.

The vectors x^* and y^*, being admissible plans for the corresponding problems, satisfy the conditions: $Ax^* = b$, $x^* \geq 0$ and $y^*A - c \geq 0$. We find the scalar product

$$(y^*A - c)x^* = (y^*A)x^* - cx^* = y^*(Ax^*) - cx^* = y^*b - cx^*.$$

According to the first theorem, the optimal values of the objective functions of mutually dual problems coincide, i.e., $y^* b = cx^*$. The latter means that $(y^*A - c)\ x^* = \sum_{j=1}^{n}\left(\sum_{i=1}^{m} a_{ij}y_i^* - c_j\right)x_j^* = 0$. However, the scalar product of two non-negative vectors can be equal to zero only if all pairwise products of their respective coordinates are equal to zero. Therefore, if $x^*> 0$, then $\sum_{i=1}^{m} a_{ij}y_i^* - c_j = 0$, if $x_j^* = 0$, then it is possible that $\sum_{i=1}^{m} a_{ij}y_i^* - c_j \geq 0$, which is stated in the theorem.

Comment. This statement can be formulated as follows. To make plans x^* and y^* are pairs of dual canonical problems (D, f) and (D^*, f^*) were optimal, it is necessary that the following conditions are met:

$$x_j^*\left(\sum_{i=1}^{m} a_{ij}y_i^* - c_j\right) = 0,$$

$$y_i^*\left(\sum_{j=1}^{n} a_{ij}x_j^* - b_i\right) = 0.$$

From these conditions it follows that if any inequality in the system of constraints in one of the canonical problems does not turn into strict equality by the optimal plan of this problem, then the corresponding element of the optimal plan of the dual problem must be equal to zero. If any element of the optimal plan of one of the canonical problems is positive, then the corresponding restriction in the dual problem of its optimal plan must turn into strict equality:

If $\sum_{i=1}^{m} a_{ij}y_i^* < c_j, j = 1,2, \dots n$ then $x_j^* = 0$

If $x_j^* > 0$ then $\sum_{i=1}^{m} a_{ij}y_i^* - c_j,\ j = 1,2, \dots n$

if $\sum_{j=1}^{n} a_{ij}x_j^* < b_i, i = 1,2, \dots, m,$ then $= 0$

if $y_i^* > 0$ then $\sum_{j=1}^{n} a_{ij}x_j^* < b_i i = 1,2,\dots,m.$

The practical significance of the duality theorems is that they allow us to replace the process of solving the main problem with the solution of the dual, which in certain cases can be simpler. For example, a problem whose range of admissible values is described by two equations connecting six variables (m = 2, n = 6) cannot be solved graphically. However, this method can be applied to solve the dual problem to it, which has only two variables.

7.7.3. Economic interpretation dual LP problem.

The traditional economic interpretation of the dual LP problem is based on *the model of the simplest production planning problem* described by us in Example 2, Section 1.4. These tasks can be economically formulated as follows.

Straight task: how many and what products x_i (i = 1, 2, ..., n) need to be produced so that for a given unit cost of production c_i, the amount of available resources b_j (j = 1, 2, ..., m) and resource consumption rates a_{ij} maximize production in value terms.

Dual task: what should be the assessment of the unit of each resource y_j (j = 1, 2, ..., m), so that for given b_j, c_i and a_{ij} minimize the overall cost estimate for all resources.

7.8. Concept of integer LP task.

Many LP problems are characterized by the fact that the variables x_j of the objective function (due to various objective properties, for example, see Section 7.3) can take only integer values. The integer linear programming problem in canonical form has the form:

$$f(x) = \sum_{j=1}^{n} c_j \times x_j \to max,$$

$$D = \left\{ x \in R^n \middle| \sum_{j=1}^{n} a_{ij} \times x_j = b_i, i \in 1{:}m\,, x_j \in Z_0^+ \right\},$$

where Z_0^+= (0, 1, 2, ...) is the set of non-negative integers.

The fundamental difficulty caused by the existence of integer conditions in the system of constraints of the optimization problem is that in a significant number of cases it is impossible to replace the discrete problem with its continuous analogue and, finding the appropriate solution, round its components to the nearest integer values. The example shown in fig. 21, demonstrates that when rounding the optimal plan x^* of the ordinary LP problem to integer values, a point is obtained that does not belong to the region of admissible plans of problem D.

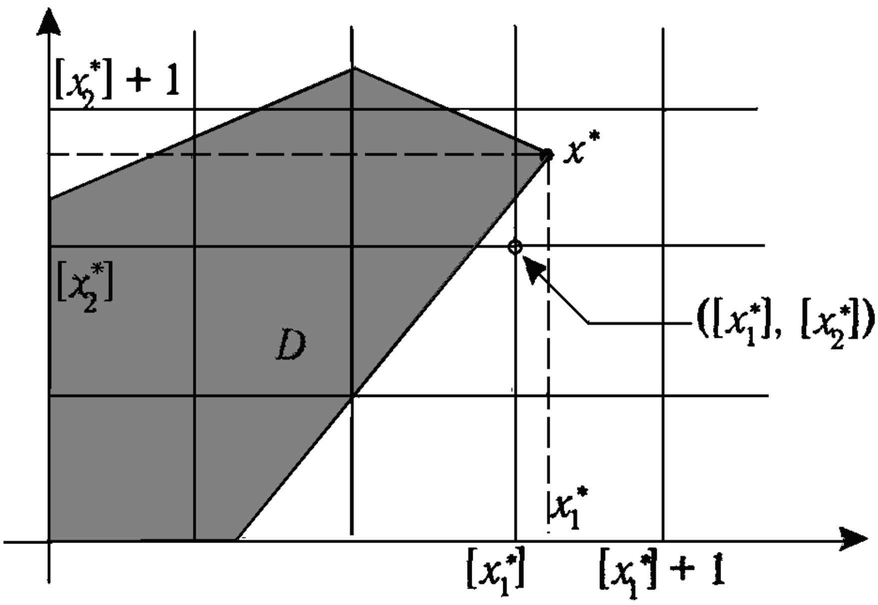


Fig. 21

Here $[x^*]$ is the integer part of x^*.

Separately, it should be added that even if the optimal plan of the continuous task, rounded to the integer values of the components, is admissible, the objective function can behave in such a way that its value will be significantly “worse” on it than on the optimal plan of the integer task.

The listed problems predetermined the need to develop special methods for solving integer problems. We invite the reader to familiarize themselves with these methods in the specialized literature.

7.9. Transport problem and methods for solving it.

Simplex is a method that we examined in previous lectures, is universal in nature and is designed to solve a very wide range of LP problems. The price for such versatility is often a decrease in its effectiveness, expressed in slow convergence, a high amount of computation, etc. At the same time, there are classes of problems for which, due to their specificity, simpler solution methods have been developed. The class of such tasks includes transport and network tasks, which are discussed in the next section of this manual.

7.9.1. Transport problem and its properties.

Let us return to the *balanced transport problem in the matrix formulation*, which we already mentioned in section 6.2, example 3 when considering the construction of mathematical models. Recall the statement of the balanced transport problem in the matrix formulation:

There are *m* departure points $A_1, A_2, \dots, A_m$, each of which contains a certain amount of cargo $(a_1, a_2, \dots, a_m)$. The load is homogeneous, size a_i called *stock.* There are *n* destinations $B_1, B_2, \dots, B_n$ where a quantity $(b_1, b_2, \dots, b_n)$ of cargo is required; value b_j – *request*. Known cost of transportation – c_{ij} from the *i*-th point to the *j*-th point (the cost of transportation may also include the cost of cargo).

The initial data of the transport problem is usually written in the form of a table, for example, Fig. 22.

It is required to draw up such a plan for the transportation of goods so that the reserves a_i are completely exhausted, applications b_j are fully executed, and the total cost of transportation would be minimal.

c_{11}	c_{12}	…	c_{1n}	a_1
c_{21}	c_{22}	…	c_{2n}	a_2
…	…	…	…	…
c_{m1}	c_{m2}	…	c_{mn}	a_m
b_1	b_2	…	b_n	

Fig. 22

Variables (unknowns) of the transportation problem (plan) are $x_{ij} \geq 0, i = 1,2, \dots, m; j = 1, 2, \dots, n$ – volumes of traffic from each *i*-th supplier to the *j*-th consumer, written in the form of a transportation matrix:

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix}.$$

This task is reduced to determining such an optimal transportation plan $x^* = (x_{11}, \dots, x_{1n}, x_{21}, \dots, x_{2n}, \dots, x_{i1}, \dots, x_{in}, \dots, x_{m1}, \dots, x_{mn})$ considered as a vector from a space whose coordinates are equal $R^{m \cdot n}$ amount of cargo x_{ij} transported from points A_i into points B_j that minimizes the objective function $f(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}$:

$$f(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij} \to min, x \in D, \qquad (7.33)$$

on the set of valid plans

$$D = \left\{ x \in R^{m \cdot n} \middle| \begin{array}{c} \sum_{i=1}^{m} x_{ij} = b_j, j = 1,2, \dots, n; \\ \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2, \dots, m; \\ x_{ij} \geq 0, i = 1,2, \dots, m; j = 1,2, \dots, n. \end{array} \right\} \qquad (7.34)$$

subject to balance conditions

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j. \qquad (7.35)$$

This mathematical model of a balanced transport problem is called *closed.*

If the task of transportation of goods is unbalanced, for example, one of the systems of restrictions of the set of permissible plans (7.34) is the inequality

$$D =$$

$$= \left\{ x \in R^{m \cdot n} \middle| \sum_{i=1}^{m} x_{ij} \geq b_j, j = 1,2, \dots, n; \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2, \dots, m; x_{ij} \geq 0 \right\},$$

or

$$D = \\ = \left\{ x \in R^{m \cdot n} \middle| \sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,\dots,n; \sum_{j=1}^{n} x_{ij} \le a_i, i = 1,2,\dots,m; x_{ij} \ge 0 \right\},$$

and therefore, the balance condition is not met: $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$, or $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$, then her mathematical model is called *open*. In this case, the unbalanced task of transporting cargo is reduced to a balanced transport task by adding, respectively, a fictitious consumer with a missing supply volume and zero transportation cost or a supplier with an excess product volume. Distribution of supply to the fictitious consumer (supplier) is the last. Therefore, in the future, by the *transport problem* (the word *balanced* is omitted) we will understand the problem of organizing the transportation of a certain product between the points of its production A_i, the number of which is equal to *m*, and *n* points of consumption B_j is the mathematical model given by expressions (7.33), (7.34) and (7.35) , i.e., is balanced.

The transport problem is a representative of the class of linear programming problems and therefore possesses all the qualities of linear optimization problems, but at the same time it also has a number of additional useful properties that made it possible to develop special methods for solving it. Features of the transport task model:

- the presence of a balance condition leads to the fact that the transport problem always has a solution;
- coefficients for variables x_{ij}, $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ in the system of constraint equations, all are equal to unity;
- the constraint matrix of the problem will have the dimension $(m + n) \times mn$, here $m + n$ is the number of all equations, *mn* is the number of all variables;
- matrices of systems of equations in constraints and $\sum_{i=1}^{m} x_{ij} = b_j \sum_{j=1}^{n} x_{ij} = a_i$ have ranks equal to respectively *n* and *t*;

- the rank matrix of the constraints of the transport problem is m + n – 1, and its non-degenerate basis plan contains m + n – 1 nonzero components. Indeed, if, on the one hand, we sum the equations $\sum_{j=1}^{n} x_{ij} = a_i$ in m, and on the other hand, the equations $\sum_{i=1}^{m} x_{ij} = b_j$ in n, then, due to the balance condition, we get the same value. It follows that one of the equations in the constraint system is a linear combination of the others. Thus, the rank of the matrix of the transport problem is m + n – 1, and its non-degenerate basic plan must contain m + n – 1 nonzero components;
- the number of basic and free variables are equal m + n – 1 and mn – (m + n – 1) = (m – 1) (n – 1), respectively. Since in the optimal solution all free variables should be equal to zero, this means that most routes will not be involved. If m = 100; n is 100; all routes (variables) 10 000, but no more than 199 will be involved.

The process of solving the transport problem is conveniently arranged in the form of a sequence of tables, the structure of which is shown in Fig. 23. It performs all the necessary calculations.

The lines of the *transport table* correspond to the points of production (the last cell of each line indicates the amount of product stock a_i), and the columns to consumption points (the last cell of each column contains the demand value b_j). All cells of the table (except those located in the bottom row and right column) contain information about the transportation from the i-th point to the j-th: in the upper left corner is the price of transportation of a product unit c_{ij}, and in the lower right – the value of the volume of transported cargo x_{ij} for these items. Cells that contain zero traffic ($x_{ij} = 0$) are called *free*, and non-zero ($x_{ij} > 0$) are called *occupied*.

c_{11} x_{11}	c_{12} x_{12}	…	c_{1n} x_{1n}	a_1
c_{21} x_{21}	c_{22} x_{22}	…	c_{2n} x_{2n}	a_2
…	…	…	…	…
c_{m1} x_{m1}	c_{m2} x_{m2}	…	c_{mn} x_{mn}	a_m
b_1	b_2	…	b_n	

Fig. 23

7.9.2. Building the initial feasible plan in the transport task.

By analogy with other linear programming problems, the solution to the transportation problem begins with the construction of an acceptable basic plan, which is further optimized. The easiest way to find it is based on the so-called *northwest corner method*. The essence of the method is the sequential distribution of all stocks available in the first, second, etc. production points, according to the first, second, etc. consumption points. Each distribution step comes down to an attempt to completely exhaust stocks at the next point of production or to an attempt to fully satisfy the needs of the next point of consumption. At each step q, the values of current unallocated stocks are indicated $a_i^{(g)}$, and current unmet needs – $b_j^{(g)}$. Building an acceptable initial plan according to the northwest corner method $a_i^{(0)} = a_i b_j^{(0)} = b_j$. For the next cell located in row i and column j, the values of unallocated stock in the i-th production point and unmet need of the j-th consumption point are considered, the minimum is selected from them and assigned as the transportation volume between these points: $x_{ij} = min\left\{a_i^{(g)}, b_j^{(g)}\right\}$. After this, the values of unallocated stock and unmet need for the relevant items are reduced by this value:

$$a_i^{(g+1)} = a_i^{(g)} - x_{ij}, \quad b_j^{(g+1)} = b_j^{(g)} - x_{ij}.$$

Obviously, at each step, at least one of the equalities holds: $a_i^{(g+1)} = 0$ or $b_j^{(g+1)} = 0$. If the first equality is true, then this means that the entire stock of the i-th production point has been exhausted and it is necessary to proceed to the distribution of stock at the production point $i + 1$, i.e., move to the next cell down the column. If $b_j^{(g+1)} = 0$, then it means that the need for the j-th point is completely satisfied, after which there follows a transition to the cell located to the right of the line. The newly selected cell becomes current, and all of the above operations are repeated for it.

Based on the condition of the balance of stocks and needs (7.35), it is easy to prove that in a finite number of steps we get an acceptable plan. By virtue of the same condition, the number of steps in the algorithm cannot be greater than $m+n-1$, therefore always remain free (zero) $mn-(m+n-1)$ cells. Therefore, the resulting plan is basic. It is possible that at some intermediate step the current unallocated stock is equal to the current unmet need $\left(a_i^{(g)} = b_j^{(g)}\right)$. In this case, the transition to the next cell occurs in a diagonal direction (the current points of production and consumption change simultaneously), which means "loss" of one nonzero component in the plan or, in other words, the degeneracy of the constructed plan.

Consider the application of the northwest corner method on a specific example. The transport table in Fig. 24 contains the conditions of a certain problem, and in the table in Fig. 25 shows the process of finding a feasible plan, including sequentially changing the amount of unallocated stocks and unmet needs. The arrows reflect the trajectory of the transition along the cells of the transport table. The table in Fig. 26 contains both the conditions of the task and the feasible plan in accordance with the table in Fig. 23.

14	28	21	28	27
10	17	15	24	20
14	30	25	21	43
33	13	27	17	

Fig. 24

27				27
6	13	1		20
		26	17	43
33	13	27	17	

Fig. 25

14 27	28 0	21 0	28 0	27
10 6	17 13	15 1	24 0	20
14 0	30 0	25 26	21 17	43
33	13	27	17	

Fig. 26

The resulting plan contains $m + n - 1 = 3 + 7 - 1 = 6$ out of 12 non-zero traffic: $x_{11} = 27$, $x_{21} = 6$, $x_{22} = 13$, $x_{23} = 1$, $x_{33} = 26$, $x_{34} = 17$ (marked in green) and, therefore, can be considered as the initial non-degenerate basic transportation plan with the value of the objective function

$$f(x) = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij} \cdot x_{ij} =$$

$$= 14 \cdot 27 + 10 \cdot 6 + 17 \cdot 13 + 15 \cdot 1 + 25 \cdot 26 + 21 \cdot 17 == 1891.$$

A feature of an acceptable plan constructed by the northwest corner method is that the objective function on it assumes a value, as a rule, far from the optimal value. This is because when it is built, transport costs c_{ij} are not taken into account in any way. In this regard, in practice, to obtain the initial plan, another method is used – the minimum element method, in which the cells with the lowest prices are primarily involved in the distribution of traffic volumes (Fig. 27).

14 27	28 0	21 0	28 0	27
10 6	17 0	15 14	24 0	20
14 0	30 30	25 13	21 17	43
33	13	27	17	

Fig. 27

The resulting plan also contains m + n – 1 = 3 + 7 – 1 = 6 out of 12 non-zero traffic: $x_{11} = 27$,,,,, $x_{21} = 6x_{23} = 14x_{32} = 13x_{33} = 13x_{34} = 17$(marked in green) and, therefore, can also be considered as the initial non-degenerate basic transportation plan, but with a lower value of the objective function

$$f(x) = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij} \cdot x_{ij} =$$

$$= 14 \cdot 27 + 10 \cdot 6 + 15 \cdot 14 + 30 \cdot 13 + 25 \cdot 13 + 21 \cdot 17 = 1720.$$

In this example, we formulate the *algorithm of the least element method*:

1. Balance the task (make sure that the task is balanced).

2. Determine the free cell with the lowest cost of transportation. If there are several such cells, then select the cell with the greatest potential cargo transportation. If there are several such cells, then select any of these cells.

3. In the selected box, put the maximum possible freight for the consumer from the supplier.

4. Check if the goods remained unallocated from this supplier.

5. If the goods are not fully distributed, then apply paragraph 2 with respect to one given supplier. Continue until the goods of this supplier are fully distributed.

If the supplier's cargo is fully distributed, check if the customer's volume is fully satisfied.

If the consumer is completely satisfied, then apply paragraph 2 regarding the remaining suppliers and the needs in the table.

If the volume of the consumer is not completely satisfied, then clause 2 applies with respect to the corresponding column.

6. Check the plan for degeneracy. The number of basic cells should be equal to $r = m + n - 1$.
7. If the plan is degenerate, then the fictitious value of the load should be set so that it is possible to find the potentials of all the base cells (set zero transportation).

8. Check the plan for optimality and, if possible, further improve it by going to the potential method.

7.9.3. Optimality criterion.

Let us consider in more detail the structure of the matrix of constraints (7.34) of the transport problem (D, f): (7.33), (7.34), (7.35). The constraint matrix has the dimension $(m + n) \times mn$, consists of zeros and ones, and splits into two groups of the same type of blocks. The first (upper group) meets the restrictions $b_j, j = 1, 2, \dots, n$ to meet the needs of consumption points, and the second (lower) – restrictions $a_i, i = 1, 2, \dots, m$ on volumes of export from production points.

We construct a dual problem (D^*, f^*). Given the specific structure of the matrix of the transport problem, the vector of dual variables will have the dimension $m + n$, and its components corresponding to the first group of restrictions are denoted by $v_j,\ j \in \overline{1, n}$ and the second through $u_i,\ i \in \overline{1, m}$. Then the dual task will have the form:

$$f^*(u, v) = \sum_{j=1}^{n} v_j b_j + \sum_{i-1}^{m} u_i a_i \to max;$$

$$D^* = \{u \in R^m, v \in R^n \big| v_j + u_i \leq c_{ij}, i \in \overline{1, m}, j \in \overline{1, n}\}. \qquad (7.36)$$

Variables u_i are called *potentials of production points* and v_j – *potentials of consumption points.*

The economic meaning of the variables of the dual task:

u_i – conditional assessment of the *i*-th supplier (conditional payment of the supplier to the carrier);

v_j – conditional assessment of the *j*-th consumer (conditional payment of the consumer to the carrier).

Applying the duality theorems (see Section 2.7.2, the third theorem), we can obtain the optimality criterion for the plan of the transport problem:

►*In order to allow a valid transport task plan* x_{ij} *was optimal, it is necessary and sufficient to find such potentials* u_i, v_j *for which*

$$v_j + u_i = c_{ij}, \text{если } x_{ij} > 0, \qquad (7.37)$$

$$v_j + u_i \leq c_{ij} \text{ или } \Delta_{ij} = v_j + u_i - c_{ij} \leq 0, \text{если } x_{ij} = 0. \qquad (7.38)$$

These conditions have a meaningful economic interpretation. According to condition (7.37), for the optimality of the transportation plan, it is required that on the routes along which the cargo is actually transported ($x_{ij} > 0$), the payment for transportation by the producer and the consumer was equal to the cost of transportation, and where these conditions are not met, the *evaluation* Δ_{ij} of the criterion (7.38) is less than zero, cargo is not transported ($x_{ij} = 0$).

It should be noted that the choice of the sign of dual variables is arbitrary and their economic interpretation may differ from the above. In most cases, the components corresponding to the first group of restrictions are denoted by $v_j, j \in \overline{1,n}$, and the second through $(-u_i), i \in \overline{1,m}$ (in the future we will adhere to these notation). Then the dual problem and the optimality criterion for the plan of the transport problem will have the form:

$$f^*(u,v) = \sum_{j=1}^{n} v_j b_j - \sum_{i-1}^{m} u_i a_i \rightarrow max,$$

$$D^* = \{u \in R^m, v \in R^n | v_j - u_i \leq c_{ij}, i \in \overline{1,m}, j \in \overline{1,n}\}; \qquad (7.39)$$

$$v_j - u_i = c_{ij}, \text{if } x_{ij} > 0, \qquad (7.40)$$

$$v_j - u_i \leq c_{ij} \text{ or } \Delta_{ij} = v_j - u_i - c_{ij} \leq 0, \text{if } x_{ij} = 0. \quad (7.41)$$

These conditions of the optimality criterion (7.40) and (7.41) have the following meaningful economic interpretation. Potentials u_i and v_j can also be considered as prices for the transported goods at the points of production and consumption (this, by the way, explains why it was necessary to designate the corresponding dual variable by $(-u_i)$). Then, according to condition (7.40), for the optimality of the transportation plan, it is required that on the routes along which the cargo is actually transported, its price at the point of consumption increase exactly by the price of its transportation, and in accordance with condition (7.41), in the optimal plan, the price of the cargo at the point of consumption cannot be less than its price at the point of production, taking into account the cost of transportation.

7.9.4. The potential method algorithm for the transport problem.

The criterion (7.37) – (7.38) or (7.40) – (7.41) is the basis of one of the methods for solving the transport problem, called the *method of potentials*. It was first proposed in 1949 by L.V. Kantorovich and M. K. Gavurin. Later, based on the general ideas of linear programming, a similar method was proposed by J. Danzig.

Just as the transport problem is a special case of the LP problem, the method of potentials, generally speaking, can be interpreted as a variety of simplex procedures. It is an iterative process, at each step of which some current basic plan is considered, its optimality is checked, and, if necessary, the transition to the "best" basic plan is determined.

The algorithm begins with the selection of some feasible basic plan (methods for constructing it were discussed in Section 7.9.2). If the given plan is not degenerate, then it contains $m + n - 1$ nonzero base cells, and one can determine the potentials from it u_i and v_j so that for each base cell (i.e., for one in which $x_{ij} > 0$) condition (7.40) was satisfied: $v_j - u_i = c_{ij}$, if $x_{ij} > 0$.

Since system (7.40) contains $m + n - 1$ equations and $m + n$ unknown, then one of the potentials can be set arbitrarily (for example, to equate v_1 or, u_1 to zero). After that, the rest are unknown u_i and v_j defined uniquely.

Consider the process of determining the potentials of the current plan of the transport problem using an example. In the table in fig. 28, the conditions of the problem and its admissible basic plan constructed by the northwest angle method are rewritten (see section 7.9.2, the table in Fig. 26).

The potential of the first point of consumption is taken equal to zero ($v_1 = 0$). Now, knowing him, we can determine the potentials for all production points associated with the first point with non-zero transportation. In this case, there are two of them (these are the first and second points), we get: $u_1 = v_1 - c_{11} = 0 - 14 = -14$; $u_2 = v_1 - c_{21} = 0 - 10 = -10$.

$v_1 = 0 \;\; v_2 = 7 \;\; v_3 = 5 \;\; v_4 = 1$

$u_1 = -14$	14 27	28 0	21 0	28 0	27
$u_2 = -10$	10 6	17 13	15 1	24 0	20
$u_3 = -20$	14 0	30 0	25 26	21 17	43
	33	13	27	17	

Fig. 28

Having u_2 and considering that in the second row of the table there are still nonzero components x_{22} and x_{23}, can determine $v_2 = u_2 - c_{22} = -10 + 17 = 7$ and $v_3 = u_2 - c_{23} = -10 + 15 = 5$ then it becomes possible to calculate $u_3 = v_3 - c_{33} = 5 - 25 = -20$ and, finally, $v_4 = u_3 - c_{34} = -20 + 21 = 1$. As a result, we obtain the complete system of potentials shown in the table in Fig. 28.

For free cells of the transport table in Fig. 28 values $\alpha_{ij} = v_j - u_i$ are calculated called *potential differences* (or the *estimate* $\Delta_{ij} = \alpha_{ij} - c_{ij}$) and are entered in the table. In the table in Fig. 29 potential differences α_{ij} written out for all non-basal cells at prices.

Potential difference α_{ij} can be interpreted as an increase in the price of a product during its transportation from point *i* to point *j*. According to the optimality criterion (7.40) – (7.41), if all $\alpha_{ij} \leq c_{ij}$ (estimates $\Delta_{ij} \leq 0$), then the plan is optimal, otherwise, if there is at least one potential difference $\alpha_{ij} > c_{ij}$ (rating $\Delta_{ij} > 0$), then it can be improved. The process of "improvement" of the plan consists in determining the input and output cells, in which a meaningful analogy can be traced with the corresponding paragraphs of simplex procedures.

$v_1 = 0 \;\; v_2 = 7 \;\; v_3 = 5 \;\; v_4 = 1$

$u_1 = -14$	14 27	28 21	21 19	28 15	27
$u_2 = -10$	10 6	17 13	15 1	24 11	20
$u_3 = -20$	14 20	30 27	25 26	21 17	43
	33	13	27	17	

Fig. 29.

The candidate for input, obviously, can be any cell in which $\alpha_{ij} > c_{ij}$, since after entering the basis equality $\alpha_{ij} = c_{ij}$ will be ensured. For definiteness, it is usually recommended to take the cell in which the score $\Delta_{ij} = \alpha_{ij} - c_{ij} > 0$ is maximum. In the example we are considering, this will be the cell (3; 1) for which $\Delta_{13} = 6$.

The output cell is determined using the so-called *chain transformation plan*, which describes the nature of the redistribution of freight flows. In accordance with the properties of the transportation problem for a non-degenerate basic plan, in the current table it is possible to form a closed chain consisting of only vertical and horizontal links, one of the vertices of which is the selected free cell, and the rest are occupied cells. In the table in Fig. 30 shows the transformation chain of the current plan relative to the entered cells (3; 1).

$v_1 = 0$ $v_2 = 7$ $v_3 = 5$ $v_4 = 1$

$u_1 = -14$	14 27	28 21	21 19	28 15	27
$u_2 = -10$	10 [−] 6	17 13	15 [+] 1	24 11	20
$u_3 = -20$	14 20 [+]	30 27	25 [−] 26	21 17	43
	33	13	27	17	

Fig. 30

The logic of the chain-building algorithm is quite simple: “leaving” cell (3.1) in the horizontal direction, we must “stop” in that busy cell of the plan from which we can move further vertically. In this example, both cell (3; 3) and cell (3; 4) satisfy this requirement. However, the chain from (3; 4) cannot be continued further, while, moving from (3; 3) vertically to (2; 3) and further to (2; 1), we return to the original cell (3 ; 1) and form a closed cycle.

In the constructed chain, starting from the input cell (which is considered to be the first), the vertices are marked: odd ones with a "+" sign, and even ones with a "–" sign. The "+" sign marks those cells in which traffic volumes should increase (such, in particular, is the cell introduced into the plan, since it should become the base). The sign "–" those cells in which traffic is reduced in order to maintain balance. Among the many cells marked with a "–" sign, the cell with the lowest value x_{ij} is selected (we denote it θ). It becomes a candidate for withdrawal, since a decrease in traffic by a large amount can lead to negative values x_{ij} in other negative cells. Then the plan is recalculated according to the chain: the volume θ is added to the volumes of traffic in the cells marked with the "+" sign and it is subtracted from the volumes of the cells marked with the "–" sign. As a result of the input of one cell and the conclusion of another, a new basic plan is obtained, for which, at the next iteration, the steps described above are repeated.

In our example, the "–" sign marks cells (2;1) and (3;3), and $x_{21} = 6, x_{33} = 26$. Having calculated the value $\theta = \min\{x_{21}, x_{33}\} = 6$, we carry out the transformation and go to the next basic plan shown in the table in Fig. 31 .

$v_1 = 0$ $v_2 = 13$ $v_3 = 11$ $v_4 = 7$

$u_1 = -14$	14 [–] 27	28 / 27	21 [+] / 25	28 / 21	27
$u_2 = -4$	10 / 4	17 13	15 7	24 / 11	20
$u_3 = -14$	14 [+] 26	30 / 27	25 [–] 20	21 17	43
	33	13	27	17	

Fig. 31

For the newly obtained plan, the actions of the standard iteration are repeated: potentials and estimates for non-basic cells of the transport table are calculated. As you can see, the plan in fig. 2.23 is also not optimal (in the cell (1; 3) $\alpha_{ij} = 25 > c_{ij} = 21$), therefore, we again build the chain of transformation of the plan and move on to the next basic plan (Fig. 32). From the transport table in Fig. 32 shows that the resulting transportation plan

$$x^* = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{pmatrix} = \begin{pmatrix} 7 & 0 & 20 & 0 \\ 0 & 13 & 7 & 0 \\ 26 & 0 & 0 & 17 \end{pmatrix}$$

optimal, since all potential differences $\alpha_{ij} = v_j - u_i$ for non-basal cells do not exceed the corresponding prices c_{ij}. According to this plan, the optimal (lowest) value of the total cost of transportation is calculated:

$$f(x^*) = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} \cdot x_{ij} =$$

$$= 14 \cdot 7 + 21 \cdot 20 + 17 \cdot 13 + 15 \cdot 7 + 14 \cdot 26 + 21 \cdot 17 = 1565.$$

$v_1 = 0 v_2 = 9 v_3 = 7 v_4 = 7$

$u_1 = -14$	14 / 27	28 / 23	21 / 20	28 / 21	27
$u_2 = -8$	10 / 8	17 / 13	15 / 7	24 / 15	20
$u_3 = -14$	14 / 26	30 / 23	25 / 21	21 / 17	43
	33	13	27	17	

Fig. 32

Using the potential method, it is possible to solve not only transport problems when the objective function is minimized, and the dual function is maximized. Quite successfully, the potential method is also used to solve problems when the objective function is maximized and the dual function is minimized. Consider this in the following example.

Student squads a_1, a_2 and a_3 numbering respectively 70, 99 and 80 people. take part in agricultural work. For harvesting potatoes in the fields b_1, b_2, b_3 and b_4 it is necessary to allocate, respectively, 47, 59, 49 and 43 people. Performance c_{ij} the student's labor depends on the potato yield on the *j*-th field, from which *i*-th unit it is calculated in centners per person per working day. Performance is presented as a matrix.

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{pmatrix} = \begin{pmatrix} 3 & 7 & 2 & 5 \\ 2 & 3 & 4 & 6 \\ 6 & 4 & 3 & 5 \end{pmatrix}.$$

It is required:

1) to distribute students into fields so that the maximum possible amount of potatoes is harvested per working day. The distribution of students by fields is determined by the matrix

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{pmatrix};$$

2) determine how many centners of potato will be harvested from four fields with the optimal distribution of students, which is given by the vector

$x^* = (x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34}) \in R^{12}$.

Potential solution.

1. Check the task for balance. The total number of people in student groups is 249 people. for 51 people more than the required number of people for harvesting potatoes 198 people.

The task is unbalanced.

To balance the task, add a fictitious potato field, for the harvest of which you need to select $b_5 = 51$ people. The labor productivity of students in a dummy field is taken equal to 0.

We compile the initial table of the balanced problem (Fig. 33).

Student detachments	Amount people	b_1	b_2	b_3	b_4	b_5
		47	59	49	43	51
a_1	70	3	7	2	5	0
a_2	99	2	3	4	6	0
a_3	80	6	4	3	5	0

Fig. 33

We write a balanced mathematical model of direct and dual problems.

The mathematical model of a direct balanced problem:

$$f(x) =$$

$$= \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij} \cdot x_{ij} = \begin{matrix} 3x_{11} + 7x_{12} + 2x_{13} + 5x_{14} + 2x_{21} + 3x_{22} + 4x_{23} \\ +6x_{24} + 6x_{31} + 4x_{32} + 3x_{33} + 5x_{34} \to max, x \in D \end{matrix};$$

$$D = \left\{ x \in R^{15} \left| \begin{matrix} x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 70, \\ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 99, \\ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 80, \\ x_{11} + x_{21} + x_{31} = 47, \\ x_{21} + x_{22} + x_{23} = 59, \\ x_{31} + x_{32} + x_{33} = 49, \\ x_{41} + x_{42} + x_{43} = 43, \\ x_{51} + x_{52} + x_{53} = 51, \\ x_{ij} \geq 0, i = 1,2,3; j = 1,2,3,4,5 \end{matrix} \right. \right\}.$$

The mathematical model of the dual balanced problem:

$$f^*(u, v) = 47v_1 + 59v_2 + 49v_3 + 43v_4 + 51v_5 + 70u_1 + 99u_2 + 80u_3 \to min;$$

$$D^* = \{u \in R^3, v \in R^5 | v_j + u_i \geq c_{ij}, i = 1,2,3;\ j = 1,2,3,4,5\}.$$

3. In contrast to the transport problem, the initial basic support plan is built not by the smallest element method, but by the *largest element method*. The largest element method differs from the least element method only in item 2 (see *Least Element Method Algorithm*). To build an initial supporting basic plan, a free cell is determined not with the least cost, but with the greatest. The constructed initial reference basic plan is entered in the table in Fig. 34.

3	7 59	2	5 11	0	70
2 18	3	4 49	6 32	0	99
6 29	4	3	5	0 51	80
47	59	49	43	51	

Fig. 34

We check the plan for degeneracy: $m + n - 1 = 3 + 5 - 1 = 7$.

The number of basic cells (highlighted in green) is 7. The plan is not degenerate.

3. Check the support plan for optimality. We set $u_2 = 0$ (in this row the largest number of base cells is 3), and from the system $m + n - 1$ equations written

for basic cells of the form $v_j + u_i = c_{ij}$ we determine the values of all potentials and enter them in the table in Fig. 35:

$v_1 = c_{21} - u_2 = 2; v_2 = c_{12} - u_1 = 8; v_3 = c_{23} - u_2 = 4; v_4 = 6; v_5 = -4; u_1 = c_{14} - v_4 = -1; u_3 = c_{31} - v_1 = 4$

$v_1 = 2; v_2 = 8; v_3 = 4; v_4 = 6; v_5 = -4.$

$u_1 = -1$	3 –2	7 59	2 1	5 [–] 11	0 [+] –5	70
$u_2 = 0$	2 [–] 18	3 5	4 49	6 [+] 32	0 –4	99
$u_3 = 4$	6 [+] 29	4 8	3 5	5 5	0 [–] 51	80
	47	59	49	43	51	

Fig. 35

We calculate estimates $\Delta_{ij} = v_j + u_i - c_{ij}$ for all unfilled cells and they are also entered in table 2.3:

$\Delta_{11} = v_1 + u_1 - c_{11} = -2; \Delta_{13} = v_3 + u_1 - c_{13} = 1;$

$\Delta_{15} = v_5 + u_1 - c_{15} = -5; \Delta_{22} = 5; \Delta_{25} = -4; \Delta_{32} = 8; \Delta_{33} = 5; \Delta_{34} = 5.$

The optimality criterion for solving the problem under consideration differs from the optimality criterion for the plan of the transport problem and is formulated as follows:

►*In order to allow a valid task plan x_{ij} was optimal, it is necessary and sufficient to find such potentials u_i, v_j for which*

$$v_j + u_i = c_{ij}, \text{если } x_{ij} > 0, \qquad (7.42)$$

$$v_j + u_i \geq c_{ij} \text{ or } \Delta_{ij} = v_j + u_i - c_{ij} \geq 0, \text{if } x_{ij} = 0. \qquad (7.43)$$

The found support solution is not optimal, since for $x_{ij} = 0$ there are negative ratings, for example $\Delta_{11} = v_1 + u_1 - c_{11} = -2 < 0$.

We proceed to the construction of the next "improved" plan.

4. A free cell (1; 5) with the lowest score ($\Delta_{15} = -5$) is introduced into the new basic plan. To determine what is derived from the initial plan of the cell, we

construct a closed loop from the basic cells with the beginning and completion of it in a free cell (1; 5). In the constructed contour, starting from the input cell (which is considered the first), we mark the vertices: odd ones with the "+" sign, and even ones with the "–" sign (see Fig. 35). Among the base cells marked with a negative sign, the cell (1; 4) with the smallest value of the variable of the initial plan is selected $x_{14} = 11$. She becomes a candidate for conclusion, and her significance $x_{14} = 11$ redistribute along the vertices of the constructed contour (where "+" value $x_{14} = 11$ is added, and where "– " – subtracted). As a result, we obtain a new admissible basic plan (see Fig. 36).For the newly constructed basic plan, we return to step 3 and for it we repeat the iterative cycle of the algorithm.

5. We check the new plan for optimality. We set $u_2 = 0$ (in this row the largest number of base cells is 3), we determine the values of all potentials and enter them in the table in Fig. 36:

$v_1 = c_{21} - u_2 = 2$; $v_2 = c_{12} - u_1 = 3$; $v_3 = c_{23} - u_2 = 4$; $v_4 = 6$; $v_5 = -4$; $u_1 = c_{15} - v_5 = 4$; $u_3 = c_{31} - v_1 = 4$.

We calculate estimates $\Delta_{ij} = v_j + u_i - c_{ij}$ for all unfilled cells and they are also entered in the table in Fig. 36:

$\Delta_{11} = v_1 + u_1 - c_{11} = 3$; $\Delta_{13} = v_3 + u_1 - c_{13} = 6$; $\Delta_{14} = v_4 + u_1 - c_{15} = 5$; $\Delta_{22} = 0$; $\Delta_{25} = -4$; $\Delta_{32} = 3$; $\Delta_{33} = 5$; $\Delta_{34} = 5$.

	$v_1 = 2$	$v_2 = 3$	$v_3 = 4$	$v_4 = 6$	$v_5 = -4.$	
$u_1 = 4$	3 3	7 59	2 6	5 5	0 11	70
$u_2 = 0$	2 – 7	3 0	4 49	6 43	0 –4 +	99
$u_3 = 4$	6 + 40	4 3	3 5	5 5	0 – 40	80
	47	59	49	43	51	

Fig. 36

The found "improved" plan is also not optimal, since there are negative ratings.

We proceed to the construction of the next "improved" plan.

6. A free cell (2; 5) with the lowest score ($\Delta_{25}= -4$) is introduced into the new basic plan. To determine which is derived from the initial plan of the cell, we construct a closed loop with the beginning and completion of it in a free cell (2; 5). In the constructed contour, starting from the input cell (which is considered the first), we mark the vertices: odd ones with the “+” sign, and even ones with the “–” sign (see Fig. 36). Among the base cells marked with a negative sign, the cell (2; 1) with the smallest value of the variable of the initial plan is selected $x_{21} = 7$. She becomes a candidate for conclusion, and her significance $x_{21} = 7$ redistribute along the vertices of the constructed contour (where "+" value $x_{21} = 7$ is added, and where "– " – subtracted). As a result, we obtain a new admissible basic plan (see Fig. 37).

$v_1 = 6$ $v_2 = 7$ $v_3 = 4$ $v_4 = 6$ $v_5 = 0$

$u_1 = 0$	3 3	7 59	2 2	5 1	0 11	70
$u_2 = 0$	2 4	3 4	4 49	6 43	0 7	99
$u_3 = 0$	6 47	4 3	3 1	5 1	0 33	80
	47	59	49	43	51	

Fig. 37

For the newly constructed basic plan, we return to step 3 and for it we repeat the iterative cycle of the algorithm.

7.We check the new plan for optimality. We set $u_2 = 0$ (in this row the largest number of base cells is 3), we determine the values of all potentials and enter them in the table in Fig. 37:

$v_1 = c_{31} - u_3 = 6$; $v_2 = c_{12} - u_1 = 7$; $v_3 = c_{23} - u_2 = 4$; $v_4 = 6$; $v_5 = 0$; $u_1 = c_{15} - v_5 = 0$; $u_3 = c_{35} - v_5 = 0$.

We calculate estimates $\Delta_{ij}= v_j + u_i - c_{ij}$ for all unfilled cells and they are also entered in the table in Fig. 37:

$\Delta_{11}= v_1 + u_1 - c_{11} = 3;\quad \Delta_{13}= v_3 + u_1 - c_{13} = 2;\quad \Delta_{14}= v_4 + u_1 - c_{15} = 1;$ $\Delta_{21}= 4; \Delta_{22}= 4; \Delta_{32}= 3; \Delta_{33}= 1; \Delta_{34}= 1.$

The found basic plan is optimal, since there are no negative ratings.

6. We determine the value of the objective function for the optimal plan

$$x^* = (x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34}) =$$
$$= (0,59,0,0,0,0,49,43, 47,0,0,0)$$

and optimal potentials

$$v^* = (v_1, v_2, \text{v}_3, v_4, v_5) = (6,7,4,6,0), \qquad u^* = (u_1, u_2, u_3) = (0,0,0)$$

direct and dual tasks:

$$f(x^*) = 7 \cdot 59 + 4 \cdot 49 + 6 \cdot 43 + 6 \cdot 47 = 1149;$$
$$f^*(u^*, v^*) = 47 \cdot 6 + 59 \cdot 7 + 49 \cdot 4 + 43 \cdot 6 = 1149.$$

Based on the first duality theorem $f(x^*) = f^*(u^*, v^*)$ the last plan is optimal.

Decision analysis. 1. In order for the maximum possible amount of potatoes to be harvested during the working day, students should be distributed among the fields as follows:

- from the squad a_1 allocate 59 people for harvesting potatoes on the field b_2, and 11 people leave in the squad;

- from the squad a_2 allocate 49 people for harvesting potatoes on the field b_3 and 43 people for harvesting potatoes on the field b_4 and 7 people leave in the squad;

- from the squad a_3 allocate 47 people for harvesting potatoes on the field b_1, and 33 people leave in the squad.

2. With this optimal distribution of students from four fields 1,149 centners of potatoes will be harvested.

Concluding the conversation about the method of potentials, we should separately dwell on the situation of the appearance of a degenerate plan. The possibility of obtaining a degenerate plan was already noted in the description of the northwest corner method. It is easy to see that a degenerate plan can also be obtained at the stage of converting the current plan along the chain: if the same minimum value is reached at once in several cells marked with the "–" sign, then when subtracting the volume moved along the chain in the new plan, it will be less than $m + n – 1$ nonzero component. The way to overcome degeneracy in the transport problem is very simple, namely: it is proposed to supplement the current plan with the necessary number of zero cells (fictitious transportation) so that they allow you to calculate the full system of potentials, continue to act in accordance with the rules of the algorithm described above. In fact, here we are dealing with nothing more than an analog of the perturbation method for the transport problem as a special case of TLP. It is easy to come to such a conclusion if we assume that the added dummy cells contain some small volume.ε

In the examples considered above, we formulate iterative *algorithm for solving the problem by the potential method:*

1. Balance the task (make sure that the task is balanced). These tasks should be entered in the source table. Write a balanced mathematical model of direct and dual problems.

2. If the task for the objective function is solved at the minimum, and the dual at the maximum, then by the method of the smallest element construct the initial basic reference plan. If the task for the objective function is solved to the maximum, and the dual to the minimum, then the initial basic reference plan is constructed using the largest element method. The constructed initial reference basic plan should be entered in the task table. Ensure that the constructed basic support plan is non-degenerate.

3. For all basic cells of the table, create a system of $m + n – 1$ equations of the form $v_j + u_i = c_{ij}$ (or $v_j - u_i = c_{ij}$.) Select variable u_i or v_j, which corresponds to the largest number of occupied (basic) cells, equate it to zero, solve the system of equations with respect to u_i or $v_j$and find these values and put them in a table.

4. For all free cells from the optimality criterion, determine the estimated difference $\Delta_{ij} = \alpha_{ij} - c_{ij}$ where $\alpha_{ij} = v_j + u_i$ is the sum potentials (or their difference $\alpha_{ij} = v_j - u_i$) enter this data in the table and check the plan for optimality.

Optimality conditions: if the task for the objective function is solved to a minimum, and the dual to a maximum, for all free cells the condition $\Delta_{ij} \leq 0$, then the optimal plan is found. For the case when the task for the objective function is solved to the maximum, and the dual to the minimum, then in the condition the inequality sign changes to the opposite: $\Delta_{ij} \geq 0$. If the indicated inequality is not satisfied for at least one cell, then it is necessary to improve the basic reference plan. The process of "improving" the plan is to determine the input and output cells in the basic plan.

5. If the task for the objective function is solved at the minimum, and the dual at the maximum, then a free cell is introduced into the new basic plan, where the estimate is positive: $\Delta_{ij} = \alpha_{ij} - c_{ij} > 0$ and maximum. If there are several such cells, then any one is selected. If the task for the objective function is solved to the maximum, and the dual to the minimum, then a free cell is introduced into the new basic plan, where the estimate is negative: $\Delta_{ij} = \alpha_{ij} - c_{ij} < 0$ and minimal. If there are several such cells, then any one is selected.

6. The output cell is determined using the chain transformation plan. In the table, you need to form a closed loop, consisting only of vertical and horizontal links, one of the vertices of which is the selected free cell, and the rest are occupied (basic) cells. “Leaving” the selected free cell, we must return to the original cell and form a closed cycle, while the circuit changes direction only in the base cells. and there should be an even number of vertices in the row and column.

7. In the constructed contour, starting from the input cell (which is considered the first), the vertices are marked: odd ones with the “+” sign, and even ones with the “–” sign. Among the cells marked with a negative sign, the cell with the lowest variable value x_{ij} is selected basic plan. She becomes a candidate for

conclusion, and her significance $x_{ij} = \theta$ redistributed over the remaining variables of the basic plan.

8. By redistributing the values along the vertices of the constructed contour $x_{ij} = \theta$ in the cell that is derived from the baseline. A new valid baseline θ is being built. Redistribution θ along the vertices of the contour according to the following rule: in the cells marked with the "+" sign, the value θ is added to the value of the baseline variable, and in the cells marked with the "–" sign, the value is subtracted. As a result, the cell deduced from the basic plan becomes free, and the first cell of the contour becomes basic with the value of the variable of the basic planθ.

9. For the newly obtained basic plan, as a result of entering one cell and outputting another, we return to step 3 of the algorithm and the actions of the standard iteration are repeated until we obtain the optimal basic plan of the transport problem.

CONTROL QUESTIONS TO TOPICS

TOPIC 1. MODERN SCIENTIFIC AND TECHNICAL PRESENTATION OF INFORMATION

1. Expand the modern content of the concept of "information" and what it is for.
2. Give a scientific definition of information and describe the process of transmitting information by a communication system.
3. Expand the meaning of the concept of “context method”.
4. Explain terms such as “information process” and disclose the features of the information process in computer technology.
5. Units for measuring the volume of text and video information in computer systems.
6. List and explain the basic properties of the information.
7. Describe the organizational and logical structure of information: data, their media and processing.
8. Describe the basic typical operations with data.

TOPIC 2. ORGANIZATIONAL AND METHODOLOGICAL FOUNDATIONS FOR THE CREATION AND FUNCTIONING OF INFORMATION SYSTEMS

9. What is a systematic approach to the analysis of information processes?
10. In what two understandings is the term “system” used and how do you define the term “system” in these terms?
11. Information systems, their structure and purpose.
12. List and characterize the main subsystems for ensuring the operation of the information system of any purpose?
13. What is necessary to ensure the operation of an information system for any purpose?
14. What is meant by information resources and information technologies of information systems?
15. What are the characteristics of the classification of information systems?
16. Computer information systems, their classification, advantages and disadvantages.

17. Software and hardware for computer information technologies and their purpose.

18. What is meant by computer program and the main list of requirements for them.

19. The principles of design and creation of computer information systems.

20. Which subsystems include corporate information systems and their purpose?

21. What is understood as a problem and its solution for an economic object with the help of corporate information system?

TOPIC 3. MANAGEMENT INFORMATION SYSTEMS

22. What are the methods of making managerial decisions?

23. What is a control system, and what are the functions of a control subsystem?

24. Describe the communication scheme for transmitting information in the control system.

25. What is the formalization of management information processes for their implementation in computers?

26. What are the characteristics of the classification of management information processes?

27. Describe the generalized scheme of information processing in control systems.

28. What subprocesses can structurally represent the information process in the control system?

29. What is the essence and purpose of information analysis and synthesis of the control system, what are their stages and content?

30. What is the content of the solutions to the following tasks of the analysis of control systems: determination of the functional features of the system; study of the information characteristics of the system?

31. What is the procedure for constructing a conceptual model of a controlling information system?

32. Describe the structure of the management information system of the self-organizing system and explain its elements.

33. What is the essence and purpose of parametric and analysis and synthesis of the control system, what are their stages and content?

34. The operator as a model for describing the concept of input - output of the "black box".

35. What is a "linear operator"? Under what conditions can a black box be considered as a linear operator? Give examples.

36. The concept of "feedback" in management information system.

37. The mathematical description of "feedback": the linear case is the Malthusian model.

38. Nonlinear Feedback - Verhulst Model.

39. Describe the stages of the operational project for the study and improvement of the management system.

40. How is the performance of the management information system evaluated to achieve a goal that can be expressed by a single criterion?

41. Give a diagram of operational research production planning.

42. What is the purpose of modeling in systems analysis, and what points should you pay attention to when modeling?

43. Illustrate graphically the entire simulation cycle.

44. What is a mathematical model and its purpose?

45. Explain the content of a scientific concept in term "Target function of the system" and its purpose.

46. What is the point of optimizing the mathematical model of the system.

47. What is the information security of computer control systems.

TOPIC 4. ECONOMIC INFORMATION PROCESSING SYSTEMS AND THEIR ROLE IN THE MANAGEMENT OF ECONOMIC OBJECTS

48. Economic object – as an object of management.

49. Imagine the economic object – the enterprise as a management system and give it a description.

50. Explain such terms as "economic object", "enterprise".

51. Explain the term "economic information", its features and properties.

52. Economic information – as a necessary element of managing economic objects.

53. An economic object is a system, its characteristics, and what is the concept of managing it.

54. List and explain the main tasks of managing economic systems.

55. Describe the structure of the management system of economic systems and explain its elements.

56. Describe the structure of the economic object management system - the enterprise.

57. What is the information system of the economic system management system, its structure, goals and objectives.

58. Define the structure of the enterprise information system.

59. What is the concept of feedback in economic systems.

60. A systematic approach to the design of information economic systems.

61. What is understood by the problem and its solution for the economic system?

62. What is a systematic approach for analyzing an economic object?

63. How is enterprise feedback organized?

64. Give a definition of the concept of "computer information system of the enterprise."

65. What are the goals and objectives of the accounting and analysis system?

66. What characteristic features are inherent in a modern accounting system?

67. What are the main requirements for accounting information, and what are its characteristics?

68. What is meant by formalization of accounting information?

69. What is the logical structure of recorded accounting information in an automated accounting system?

70. By what principle is accounting information classified in a database in an automated accounting system?

71. In what form are data organized and stored in a database in an automated accounting system?

72. What role does the classifier play in an automated accounting system?

73. What are the main classification systems for accounting information in an automated accounting system and what is the difference between them?

74. What are the advantages and disadvantages of the hierarchical system for classifying accounting information in an automated accounting system?

75. What are the advantages and disadvantages of the offset system for the classification of accounting information in an automated accounting system?

76. What kind interpretation models of data accounting documents are used in automated accounting system?

77. What is accounting information, and what is its role in the management system?

78. What are the signs of classification of economic information, its meaning and use.

79. What are the main signs of the classification of computer accounting programs.

80. What is the difference between the classification of accounting information from its encoding?

TOPIC 5. SOFTWARE ECONOMIC INFORMATION PROCESSING SYSTEMS

81. List and characterize the basic services provided by banks using information technology and telecommunication environment.

82. List and characterize the main types of virtual banking services.

83. Give a comparative description of investment analysis software.

84. Give a comparative description of financial analysis software products in enterprises.

85. Describe the general information system of the pension fund.

86. What are the main functions of the "Appointment and payment of pensions and benefits" software?

87. Give a characteristic enterprise management software products: enterprise demand planning systems for MRP materials and all MRPII resources, ERP accounting, planning and control systems, CRM customer relationship management systems.

88. What business performance indicators can be improved using MRP information systems, MRPII, ERP and CRM Bitrix24?

89. Perform an analysis programs for managing documents and documents – **MEDOC.**

90. What is the reason for the daily life and purpose of digital information technology? the module "Electronic Document Management" programs MEDOC.

91. Describe the software security management process ELMA BPM.

THEME 6. ECONOMICALLY-MATHEMATICAL MODELING AS ECONOMIC MANAGEMENT DECISION MAKING METHOD

92. What is mathematical modeling and mathematical model, for what purpose is it used and their classification.
93. What is the essence of mathematical and simulation modeling of economic systems?
94. What is the procedure for constructing simulation models using a systematic approach?
95. Describe the dynamic modeling technique, proposed by Forrester, allowing you to simulate the activities of the enterprise on a computer.
96. Expand the basic concepts of "operations research" and statement of the problem of operations research.
97. Describe examples of the use of the concept of "input-output" in the modeling of economic systems. In particular, is this concept used in microeconomics? Argument the answer.
98. What is the objective function of the mathematical model of the economic system?
99. What is the principle of optimality of acceptable strategies (plans)?
100. Based on what principles is the classification of operations research tasks carried out?
101. On the basis of what principles is the construction of an optimization (operational) model of operations research tasks carried out?
102. Describe the economic-mathematical model of asset portfolio management.
103. Describe the economic and mathematical model production planning.
104. Describe the economic and mathematical model transport task.
105. Describe the economic and mathematical model use of enterprise facilities.
106. Based on what principles classification of research tasks of economic-mathematical models is carried out.

THEME 7. METHODS OF FINDING THE OPTIMAL SOLUTION FOR THE ECONOMIC-MATHEMATICAL MODEL: LINEAR PROGRAMMING

107. Formulate the linear programming problem and their geometric interpretation, give examples.

108. What is the difference between the general linear programming problem and the canonical problem?

109. How can the general linear programming problem be reduced to canonical form?

110. Give a definition for the following concepts: plan, feasible plan, optimal plan, solution of the optimization problem.

111. The basic concepts of linear algebra used in the theory of mathematical programming:

- linear space R^n and V_3 over the field R of real numbers;
- linear span of a system of vectors in the linear space of its properties and rank;
- the basis of the vector space and its properties;
- isomorphism of finite-dimensional linear spaces;
- affine set (space) and its properties;
- convex set;
- the set of solutions to a joint, indefinite, heterogeneous system of linear algebraic equations with real coefficients and n unknowns $(x_1, x_2, \dots, x_n) \in R^n$.

112. What is the first geometric interpretation of the linear problem programming?

113. What is the second geometric interpretation of the linear programming problem? What is its difference from the first?

114. What plan is called basic and the method of finding it?

115. Which plan of the linear programming problem is called degenerate, and what situation in geometric interpretation does this correspond to?

116. From the point of view of the second geometric interpretation, how can one imagine the process of finding the optimal plan in a linear programming problem?

117. Formulate the optimality criterion for an admissible basic plan used in the simplex method.

118. Formulate the main stages of the standard iteration of the simplex method.

119. What element of the simplex table is called the lead?

120. Under what conditions is it concluded that the objective function is unbounded in the problem being solved? What geometric interpretation corresponds to this case?

121. What linear programming problem is called degenerate? By what signs can you find out that the current plan is degenerate and in what way can this be eliminated?

122. Modified simplex method. Transformation of Jordan – Gauss.

123. Define a dual task and its economic interpretation.

124. Formulate duality theorems that establish the relationship between solutions of the direct and dual LP problems.

125. What are the main properties of a pair of dual tasks?

126. What is fundamental complexity caused by the presence of integer conditions in the system of constraints of the optimization problem of the LP?

127. Articulate iterative algorithm for solving the optimization LP problem by the potential method.

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