Reverse inequalities for geometric and power means

A. A. Korenovskyi

Odessa I. I. Mechnikov National University, Ukraine anakor@paco.net

Let $f: [0,1] \to \mathbf{R}$ be a non-negative function. The functions

$$M_0 f(t) = \exp(t^{-1} \int_0^t \ln f(u)\, du) \quad \text{and} \quad M_\alpha f(t) = (t^{-1} \int_0^t f^\alpha(u)\, du)^{1/\alpha}, \quad 0 < t \le 1,$$

are called geometrical and power means of order $\alpha \neq 0$, respectively. Note that the function $M_\alpha f$ is monotonically increasing in α. Fix $-\infty < \alpha < \beta < +\infty$, $B > 1$, and consider the class $RH^{\alpha,\beta}(B)$ of functions f satisfying the "reverse inequality"

$$0 < M_\beta f(t) \leq B \cdot M_\alpha f(t) < +\infty, \quad 0 < t \leq 1.$$

The main property of such classes consist in the "self-improvement" of the summability exponents of functions $f \in RH^{\alpha,\beta}(B)$. In the talk we are going to discuss a similar property. Namely, for a function $f \in RH^{0,\beta}(B)$, the boundary values for positive and negative summability exponents of the mean $M_\beta f$ are established. Analogously, for $f \in RH^{\alpha,0}(B)$ similar "critical" summability exponents for the mean $M_\alpha f$ are found.

The exact formulations of the corresponding results and their proof are presented in [1]. If $f \in RH^{\alpha,\beta}(B)$ and $\alpha \cdot \beta \neq 0$, analogous problems have been studied in [2] and [3].

This work is partially supported by the University of Brunei Darussalam under Grant UBD/GSR/S&T/19.

1. Korenovskyi A. A. *Reverse inequalities for the geometric and power means*, Ukr. Math. J. (to appear).
2. Didenko V. D., Korenovskyi A. A. *Power Means and the Reverse Hölder Inequality*, Studia Math. **207**: (2011), 85–95.
3. Didenko V. D., Korenovskyi A. A. *The reverse Hölder inequality for power means*, J. Math. Sci. **183**:6 (2012), 762–771.