ПРОЕКТУВАННЯ І МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ СЕНСОРІВ

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SENSING NON-LINEAR CHAOTIC FEATURES IN DYNAMICS OF SYSTEM OF COULED AUTOGENERATORS: STANDARD MULTIFRACTAL ANALYSIS

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Abstract

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The multifractal approach has been used for analysis and sensing the non-linear chaotic features in dynamics of system of the coupled autogenerators. It has been found that the corresponding fractals dimensions are lying in the interval [1,3-1,9].

Key words: sensing dynamics, system of coupled autogenerators, fractal analysis

Анотація

ДЕТЕКТУВАННЯ НЕЛІНІЙНИХ ХАОТИЧНИХ ЕЛЕМЕНТІВ У ДИНАМИЦІ ОСЦИЛЯЦІЙ В СИСТЕМІ ЗВ'ЯЗАНИХ АВТОГЕНЕРАТОРІВ: МУЛЬТІФРАКТАЛЬНИЙ АНАЛІЗ

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Мультіфрактальний підхід використано для аналізу та детектування хаотичних елементів у динамиці осциляцій в системі зв'язаних автогенераторів. Відповідний спектр фрактальних розмірностей лежить у інтервалі [1,3-1,9].

Ключові слова: детектування, динаміка, система зв'язаних автогенераторів, фрактальний аналіз

Аннотация

ДЕТЕКТИРОВАНИЕ НЕЛИНЕЙНЫХ ХАОТИЧЕСКИХ ЭЛЕМЕНТОВ В ДИНАМИКЕ КОЛЕБАНИЙ В СИСТЕМЕ СВЯЗАННЫХ АВТОГЕНЕРАТОРОВ: МУЛЬТИФРАКТАЛЬНЫЙ АНАЛИЗ

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Мультифрактальный подход использован для анализ и детектирования элементов хаоса в динамике колебаний в системе связанных автогенераторов. Соответствующий спектр фрактальных размерностей лежит в интервале [1,3-1,9].

Ключевые слова: детектирование, динамика, система связанных автогенераторов, фрактальный анализ

1. Introduction

Many physical and biological systems — multielement semiconductors and gas lasers, different radiotechnical devices, etc can be considered in the first approximation as set of autogenerators, coupled by different way. So, experimental and theoretical studying these non-linear dynamical systems with am aim to discover the fractal features and elements of dynamical chaos is of a great importance the (c.f.[1-14]). The typical scheme of two autogenerators (semiconductore quantum generators (1), coupled by means optical waveguide (2), is presented in figure 1. An important feature of these systems is connected with possibility of realizing so called sinphase regimes of autooscillations, when relative phases of oscillations of different elements are fixed. Another important feature is realizing the stochastic regime of oscillations and chaos elements.

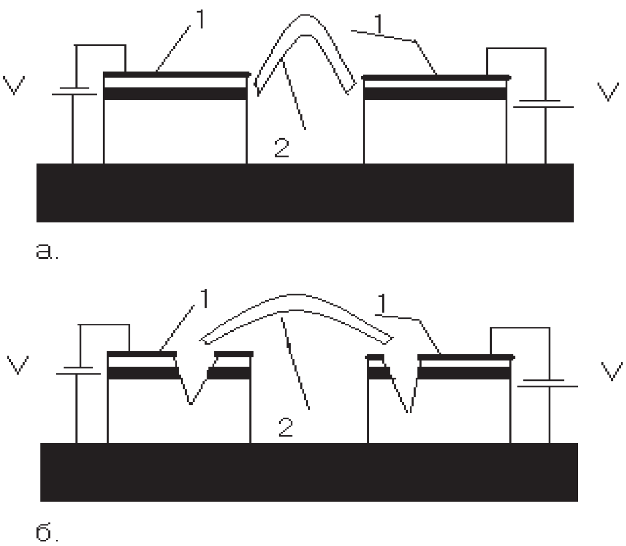

Fig. 1. Scheme of two autogenerators (semiconductore quantum generators (1), coupled by means optical waveguide (2).

In ref.[4,8] it has been numerally studied a regular and chaotic dynamics of the system of the Van-der-Poll autogenerators with account of a finiteness of signals propagation time between them and also with special kind of interaction forces between the oscillators. The cases of little and large non-linearity in the system are considered. In this paper we use the standard multifractal approach for an analysis and sensing the non-linear chaotic features in dynamics of system of the coupled autogenerators.

2. Dynamics of a system of the coupled autogenerators

The equation of motion for system of oscillating dipoles, situated in points with co-ordinates $r_i(I=1...N)$, can be written as follows: :

$$\ddot{x}_l{}^{\bullet} + \text{å}_l\left(x_l^2-\text{ã}_l\right)\dot{x}_l+\text{ù}_l^2x_l=-\sum_{l\neq l} f_{ll'}\,\ddot{x}_{l'}\left(t-\text{ô}_{ll'}\right) \quad (1)$$

where ω_i are the own frequences of autovibrations; the dipole moment vectors are directed along axe z; $d_i=(0,0,d), d\equiv e_ix_i$; e_i is an effective charge of "*i*" dipole; ε_l is parameter of non-linearity; $f_{ll'}$ is parameter of link of the generators. The force in the right part of equation (1) describes an action on "*l*:" oscillator provided by the radiation field of other oscillators. It has the following form:

$$f_{ll'}=(e_le_{l'}/m_lc^2)exp[-\delta|r_l-r_{l'}|](1/|r_l-r_{l'}|). \quad (2)$$

In the limit of $\delta\to 0$:

$$f_{ll'}=(e_le_{l'}/m_lc^2)(1/|r_l-r_{l'}|). \quad (3)$$

The last expression is corresponding to condition, when a distanse between dipoles is more than the radiation wavelength: $\lambda_l=2\pi/\omega_l(\omega_l>>\tau_{ll'}\equiv \equiv c|r_l-r_{l'}|^{-1})$. The little non-linearity condition corresponds to unequalities: $\varepsilon_l<<1$,$f_{ll}<<1$. In this case a solution can be searched in the following form:

$$x_i = a_l cos(\omega t + \varphi_l)$$

where a_l, φ_l are the slow variables. The numerical solution of problem is carried out in ref.[7]. Phase diagram for system of two coupled autogenerators, interacting with delay, is presented in figure 2. Digits on scheme show the regions, where single-frequency sinphase oscillation regime (1), multi-frequaency sinphase one (2), chaotic one (3) are realized.

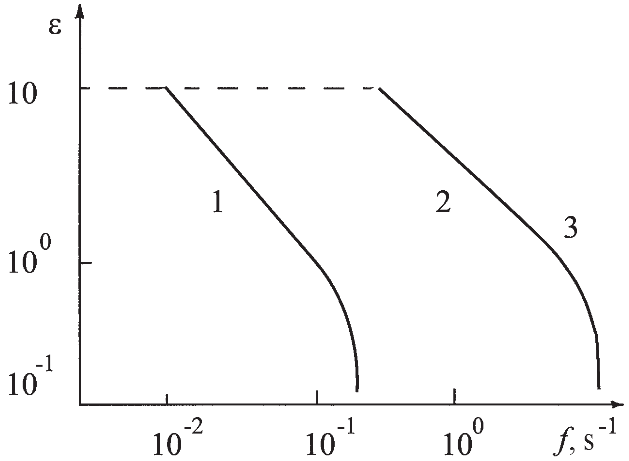

Fig.2. Phase diagram for system of two coupled autogenerators, interacting with delay. Digits on scheme show the regions, where single-frequency sinphase oscillation regime (1), multi-frequency sinphase regime (2) and chaotic regime (3) are realized.

It is important to note that an increasing the non-linearity parameter ε_l and parameter of link of the generators $f_{ll'}$ leads to a significant growth of complexity of the phase trajectory of the system. In figure 3 we present the calculated spectrum of oscillations system of two coupled autogenerators (in time $t \approx 40\tau$ after start of oscillations) [7]. In ref.[8] it was carried out analysis of oscillations in system of the coupled autogenerators in a chaotic regime (regime 3 in figs.2) within the wavelet- multifractal formalism and the fractals dimensions interval has been found. Here we use the standard version of multifractal approach (c.f.[9-11]) for analysis and sensing the non-linear chaotic features in dynamics of system of the coupled autogenerators.

3. Multifractal approach. Results

Let us remember that since last decades multifractal approach is used as the new powerful tool for analyzing and sensing various signals. At present, the multifractal approach is being increasingly used in problems of pattern recognition; in processing

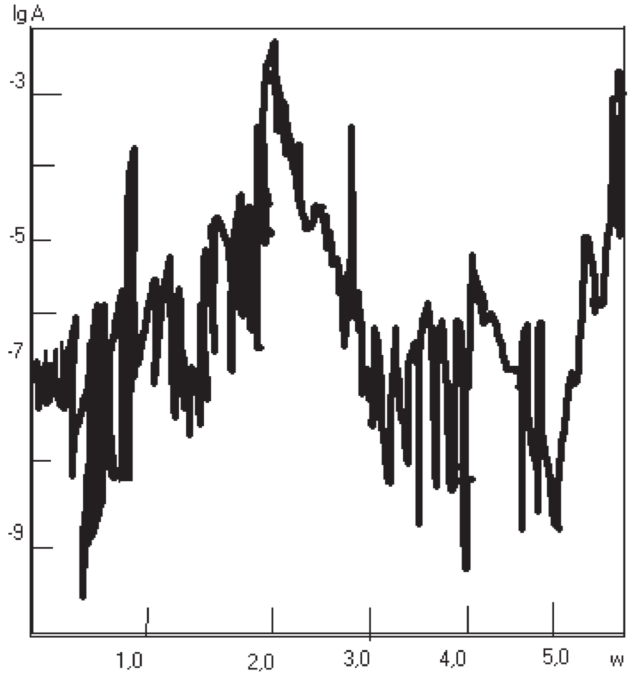

Fig.3. Spectrum of oscillations system of two coupled autogenerators, interacting with delay (in time $t \approx 40\tau$ after start of oscillations).

various signalsets, in an analysis of the images of any kind (X-ray picture of a kidney, an image of mineral, etc.); for study of turbulent fields, for contraction (compression) of large volumes of information, and in many other cases. Non-uniform and multi-fractal objects can be more completely characterized by spectrum of $D(q)$ fractal exponents, where q is a real number, the so-called generalized dimension, where the fractal dimension is equal to $D(0)$ and the function $D(q)$ is generally referred to as multifractal spectrum (c.f. [2,10]). Mathematically, the general aim of the multifractal formalism is to determinate the $f(\alpha)$ singularity spectrum of measure μ. It associates the Haussdorff dimension of each point with the singularity exponent α, which gives an idea of the strength of singularity: $N_\alpha(\varepsilon) = \varepsilon^{-f(\alpha)}$, where $N_\alpha(\varepsilon)$ is the number of boxes needed to cover the measure and ε the size of each box.. A partition function Z can be defined from this spectrum:

$$Z(q,\varepsilon) = \sum_{i=1}^{N(\varepsilon)} \mu_i^q(\varepsilon) \approx \varepsilon^{\tau(q)} \text{ for } \varepsilon \to 0$$

where $\tau(q)$ is a spectrum which arouses by Legendre transforming the $f(\alpha)$ singularity spectrum. The spectrum of generalized fractal dimensions is obtained from the spectrum $\tau(q)$:

$$D_q = \frac{\tau(q)}{(q-1)}.$$

The practical procedure for calculating a multifractal spectrum is carried out with q in some range usually from 0 to q_c. This range q is suitable for characterizing the system dynamics time-series with multifractal exponents. The corresponding time-series are generally non-linear parameter dependent and have parameter ranges, in which the dynamics is chaotic. Chaotic behaviour, in the sense of a fully deterministic evolution of the system in time, yet erratically looking behaviour, bounded in phase space with sensitive dependence on initial conditions, might therefore be expected to occur also in the time series. A realization of the chaotic regime (the region 3 in fig.2) in our system begins to take a place for parameters $\varepsilon_l>0{,}1$ and $f>0{,}3$. Our numerical analysis has shown that the fractals dimensions for oscillations in system of the coupled autogenerators in a chaotic regime are lying in the interval [1,3-1,9]. It is important to note that our data are in an excellent agreement with the wavelet fractal estimates [8]. In conclusion let us note that the data regarding to the multi-fractal spectra allow restoring and forecasting the time evolution behaviour on some necessary temporary interval. We suppose that this is one of the most effective advantanges of the multi-fractal formalism to problem of non-linear statistical analysis of the coupled autogenerators system evolution time series. In any case it is obvious that the next step in description of dynamical systems considered is in using the unity and scales invariance of the master dynamical equations which describe an evolution of the system.

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